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Stability of an overland flow scheme in the framework of a fully coupled eco-hydrological model based on the Macroscopic Cellular Automata approach

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ABSTRACT

Cellular Automata are often used for modeling the evolution in time of environmental systems mainly because they are directly compatible with parallel programming. Nevertheless, defining the optimal time step criterion for integrating forward in time numerical processes can further enhance model computational efficiency. To this aim, a numerical stability analysis of an original overland flow model, within the framework of a fully coupled eco-hydrological system based on the Macroscopic Cellular Automata paradigm, is performed. According to the other modules of the system describing soil water flow, soil-surface-atmosphere fluxes and vegetation dynamics, overland flow model equations were derived through a direct discrete formulation (i.e. no differential equations were discretized), adopting the diffusion wave model as an approximation of the full De Saint Venant equations and including the capability of accounting for specific processes, such as the increasing roughness effects due to vegetation growth or surface-soil water exchanges. Suitable formulations of robust tools usually applied in the stability analyses, such as Courant–Friedrichs–Lewy and von Neumann conditions, were initially derived for the CA-based overland flow model. Afterwards, the theoretical stability conditions were compared to experimental time step constraints through several numerical simulations of a 5-h rain event. Specifically, adopting a constant (i.e. not adaptive) time step for simulations, and discretizing head losses in a way that increases model stability, experimental upper limits preventing numerical instability were found for 13 test cases with different slopes, precipitation intensities, vegetation densities and depths of surface depressions. Even though von Neumann condition and experimental values were well positively correlated, the latter were almost always sensibly lower, excluding cases when free surface gradients tended to zero. Therefore, based on the original method, two alternative criteria were developed. Numerical tests showed that the joint use of these criteria greatly helps in finding the optimal time steps for convergent and stable simulations of the overland flow model.

1. Introduction

The extension of the original Cellular Automata (CA) definition through the introduction of the Macroscopic CA (MCA) paradigm [15,14] allowed to use this discrete approach for modeling several fluid dynamics phenomena at the macroscopic scale (e.g. [12,26,32,7,8,16,3]).

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The MCA mainly differ from the classical CA structure because an almost unlimited number of states is allowed for the cells, and every state is expressed by a Cartesian product of sub-states, each one describing a specific feature of the space portion (e.g. a finite volume of soil or fluid) related to its own cell. Nevertheless, most of the MCA applications use transition functions based on parameters whose physical meaning is not clear, so they need heavy calibration phases. The first example of MCA modeling in hydrology using a transition function based on physical laws is given by Mendicino et al. [26], who developed a three-dimensional model which simulates at the hillslope/catchment scale water flux in unsaturated soils deriving a direct discrete formulation of the field equations, through the application of elementary physical laws to single finite cells with uniform physical properties. However, at the hillslope and catchment scale several interconnected processes regulate surface–subsurface water movement and its transfer from/to the atmosphere. For example, infiltration or saturation excess can be dealt with only accounting for overland flow processes, while evapotranspiration, a fundamental component of the hydrological cycle, is strongly dependent on the energy balance at the surface. All these processes, in turn, are deeply influenced by vegetation properties [2,29], which affect, e.g., surface roughness or resistance to evaporative transfer. Furthermore, plant roots also interact with soil flux processes. With the aim of developing a complete modeling of the complex eco-hydrological processes at the hillslope and catchment scale, the MCA-based model was initially extended with two different Vegetation Dynamic Models (VDMs) coupled to a Soil–Vegetation–Atmosphere Transfer (SVAT) scheme [7,8].

One of the most challenging obstacles to the operational use of highly detailed eco-hydrological models is their huge computational burden if applied to large scales. From this point of view the option of MCA is very suitable, because it can easily lead to efficient parallel computing by means of the increasing number of multi-core processors available. However, some numerical issues such as convergence and stability problems still exist. Mendicino et al. [26] performed a complete convergence analysis finding a suitable time step criterion for their unsaturated flow model, which was recently enhanced by Anagnostopoulos and Burlando [1].

Model stability issues are even more challenging when dealing with runoff generation mechanisms and overland flow routing processes. More severe rules for time steps are usually required for several reasons, such as the typically higher speed of surface water flow with respect to subsurface flow, problems arising with very low values of flow depth and changes in state of the cells from dry to wet and vice versa. An overland flow module was added to the MCA system and is presented in this paper. The new module, intended for a highly detailed analysis at the hillslope scale, allows the system to be the first based on CA approach, increasing the rather small number of fully coupled eco-hydrological models, which are aimed at the comprehensive simulation of the hydrological processes accounting for the complex interactions with atmospheric boundary layer and vegetation (Rhessys, [34]; GeoTop, [31]; tRIBS + VEGGIE, [24]; Tethys-Chloris, [18]). Differently from other simple overland flow models, the proposed approach is able to account for specific processes, such as vegetation dynamics effects or surface-soil water exchanges. In perspective, it lays the foundations to future developments aimed at modeling surface–subsurface environmental transport processes, where interactions between water flow and solid frame can be dealt with by means of methods such as the phase average technique [40,41,39,38].

Due to the complex interactions of the overland flow model with the other elements of the system, and considering the novelty of the MCA approach, it is necessary to evaluate how classical methods for deriving stability constraints fit to numerical simulations, and possibly to develop and test model-specific alternative criteria. This is the main issue faced in the paper.

Stability problems related to overland flow equations are deeply analyzed in literature. The milestone is the so-called Courant–Friedrichs–Lewy (CFL) condition [10], a convergence condition which for a consistent finite-difference scheme becomes identical to the stability condition [17]. However, CFL condition is a kind of upper boundary to time step, often not sufficient for preventing problems like oscillatory behaviors (e.g. [5]. A more strict stability criterion is provided by the so-called von Neumann condition [9], that was successfully used inter alia by Hunter et al. [23] in their LISFLOOD-FP model, based on the diffusive approximation. Further studies showed that it is possible to relax this criterion without degrading too much (i.e. over the order of error of elevation data) models performance [20], and that more stable schemes can be achieved by re-adding part of inertial terms to the diffusive wave simplification of the De Saint Venant equation [4,16,13]. In the presented overland flow model, given the complexity of the overall coupled system, the discrete formulation adopted for deriving flow equations was as simple as possible, being based on the De Saint Venant diffusive form equation. Therefore, potential improvements in the model stability due to the inertial formulation at this stage of model development are not yet considered; however, Bates et al. [4] stated that the inertial formulation is not always as accurate as the diffusive formulation.

Summarizing, the main objectives of the paper are: (1) to describe an overland flow model integrated in the framework of a fully coupled eco-hydrological model based on the MCA paradigm; (2) to develop formulations of classical robust stability conditions like CFL or von Neumann condition that can be appropriately applied to the CA-based overland flow model and, to describe also some numerical adjustments useful for increasing model performances; (3) to quantify the improvement due to numerical adjustments and to assess the reliability of the theoretical stability conditions. These conditions will be compared to the experimental model stability constraints, by means of a series of numerical simulations where some of the main parameters of the eco-hydrological model (namely slope, precipitation input, vegetation density, mean depth of surface depressions) are gradually varied in order to assure that as many scenarios as possible are analyzed. It will be shown that in most cases the use of both CFL and von Neumann conditions does not allow model stability and two alternative criteria will be introduced and tested.

The main features of the new runoff generation and flow routing scheme are described in the next Section 2; the demonstrations for deriving the theoretical stability conditions to be applied to the CA scheme are shown in Section 3; finally, results of the numerical simulations and the two alternative criteria are shown in Section 4.