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Reprint of Infinity computations in cellular automaton forest-fire model ☆

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ABSTRACT

Recently a number of traditional models related to the percolation theory has been considered by means of a new computational methodology that does not use Cantor's ideas and describes infinite and infinitesimal numbers in accordance with the principle 'The whole is greater than the part' (Euclid's Common Notion 5). Here we apply the new arithmetic to a cellular automaton forest-fire model which is connected with the percolation methodology and in some sense combines the dynamic and the static percolation problems and under certain conditions exhibits critical fluctuations. It is well known that there exist two versions of the model: real forest-fire model where fire catches adjacent trees in the forest in the step by step manner and simplified version with instantaneous combustion. Using new approach we observe that in both situations we deal with the same model but with different time resolution. We show that depending on the "microscope" we use the same cellular automaton forest-fire model reveals either instantaneous forest combustion or step by step firing. By means of the new approach it was also observed that as far as we choose an infinitesimal tree growing rate and infinitesimal ratio between the ignition probability and the growth probability we determine the measure or extent of the system size infinity that provides the criticality of the system dynamics. Correspondent inequalities for grosspowers are derived.

1. Introduction

Recently a new applied point of view on infinite and infinitesimal numbers has been introduced in [1,19,24,26]. The new approach does not use Cantor's ideas (see [6]) and describes infinite and infinitesimal numbers that are in accordance with

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the principle 'The whole is greater than the part' (Euclid's Common Notion 5). It gives a possibility to work with finite, infinite, and infinitesimal quantities *numerically* by using a new kind of computers – the Infinity Computer – introduced in [20,21,27,28]. In our previous paper [2] we applied the new computational tools to study percolation phase transition. It has been established that in an infinite system phase transition point is not really a point as with respect of traditional approach. We showed that in light of new arithmetic it appears as a critical interval, rather than a critical point. Depending on the "microscope" we use this interval could be regarded as either finite or infinite or infinitesimal interval. Using new approach we observed that in vicinity of percolation threshold we have many different infinite clusters instead of one infinite cluster that appears in traditional consideration. Moreover, we have now a tool to distinguish those infinite clusters. In particular, we can distinguish spanning infinite clusters from embedded infinite clusters.

In this paper, we are going to apply the new arithmetic to a cellular automaton forest-fire model [10,11] which is tightly connected with the percolation methodology and in some sense combines the dynamic and the static percolation problems. Forest-fire model elegantly represents the simplest example of a model that under certain conditions exhibits critical fluctuations. Actually, community that deals with the forest-fire model and its applications, distinguishes between two versions of the model: real forest-fire model where fire catches adjacent trees in the forest in the step by step manner and a simplified version with the instantaneous combustion [10,12]. Using the new approach we show that in both situations we deal with the same model but with different time resolution. We observe that depending on "microscope" we use the same cellular automaton forest-fire model reveals either instantaneous forest combustion or step by step firing.

Another interesting feature of the forest-fire model is that the model is critical when driven in a certain limit. That is, critical behavior will occur in the limit of slowly growing trees. Though slow-growing, the trees must grow quickly compared to the time interval between spontaneously ignited fires, i.e., the ratio between the ignition probability and the growth probability should be moved toward zero. It should be emphasized that the limits discussed imply infinite model system size, so-called thermodynamic limit. Finite model size limit results in some difficulties during cellular automaton forest-fire model implementation in numerical experiment. The new computational approach proposed recently in [19,24] allows us to overcome this difficulties. As far as we choose infinitesimal trees growing rate and infinitesimal ratio between the ignition probability and the growth probability we determine the measure or extent of the system size infinity that provides the criticality of the system dynamics. By means of the new approach we derive correspondent inequalities for grosspowers that control trees growing rate infinitesimality, infinitesimal ignition probability and the system size infinity.

The outline of the paper is as follows. In Section 2 we introduce the new approach methodology that allows one to write down different finite, infinite, and infinitesimal numbers by a finite number of symbols as particular cases of a unique framework and to execute numerical computations with all of them. Then in Section 3 we briefly remind basic features of the percolation phase transition and summarize some features of infinity percolation cluster in terms of infinity computations. In Section 4 we apply the new arithmetic to the cellular automaton forest-fire model. In the final section, the application results are summarized and discussed.

2. Methodology

In this section, we give a brief introduction to the new methodology that can be found in a rather comprehensive form in [24,26,30] downloadable from [21] (see also the monograph [19] written in a popular manner). A number of applications of the new approach can be found in [7,8,16,17,22,23,25,27–32,34,35]. We start by introducing three postulates that will fix our methodological positions (having a strong applied character) with respect to infinite and infinitesimal quantities and Mathematics, in general.

Usually, when mathematicians deal with infinite objects (sets or processes) it is supposed that human beings are able to execute certain operations infinitely many times. Since we live in a finite world and all human beings and/or computers finish operations they have started, this supposition is not adopted.

Postulate 1. *There exist infinite and infinitesimal objects but human beings and machines are able to execute only a finite number of operations.*

Due to this Postulate, we accept a priori that we shall never be able to give a complete description of infinite processes and sets due to our finite capabilities.

The second postulate follows the way of reasoning used in natural sciences where researchers use tools to describe the object of their study and the used instrument influences the results of observations. When physicists see a black dot in their microscope they cannot say: The object of observation *is* the black dot. They are obliged to say: the lens used in the microscope allows us to see the black dot and it is not possible to say anything more about the nature of the object of observation until we change the instrument – the lens or the microscope itself – by a more precise one.

Due to Postulate 1, the same happens in Mathematics studying natural phenomena, numbers, and objects that can be constructed by using numbers. Numeral systems used to express numbers are among the instruments of observations used by mathematicians. Usage of powerful numeral systems gives the possibility to obtain more precise results in Mathematics and in the same way usage of a good microscope gives the possibility of obtaining more precise results in Physics. However, the capabilities of the tools will be always limited due to Postulates 1 and 2, so, we shall never tell, **what is**, for example, a number but shall just observe it through numerals expressible in a chosen numeral system.

Postulate 2. *We shall not tell* ***what are*** *the mathematical objects we deal with; we just shall construct more powerful tools that will allow us to improve our capacities to observe and to describe properties of mathematical objects.*