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Acoustic propagation in two-dimensional waveguide for membrane bounded ducts

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ABSTRACT

This work aims to investigate the mode-matching (MM) and low frequency approximation (LFA) solutions of a two dimensional waveguide problem with flanged junction. The relative merits of each approach are compared for the scattering of fluid-coupled wave. The boundary value problem involving higher order derivatives at boundaries becomes a non-Sturm–Liouville problem where the use of standard orthogonality relation (OR) enables the MM solution. The derivation of LFA is made which proves to be surprisingly accurate for structure-borne mode incident. In order to validate the truncated model expansion the distribution of power in duct regions is discussed and Gibbs oscillations are incorporated by reconstruction of the normal velocity field using Lanczos filter.

1. Introduction

The structural acoustic has become a stimulating and attention-grabbing issue of this era. The interest to minimize the ducted fan noise aero engines, power station and heating, ventilation, and air conditioning (HVAC) system offers a great inspiration to engineers and scientists. Duct work is common feature in engineering structures, air craft and buildings etc. which clearly beneficial but also a channel for unwanted noise. It propagates sound at significant distances by the mechanism of reflection through the internal walls of duct and duct vibration.

In order to reduce such noises some dissipative devices likewise expansion chamber, acoustic lining or silencer and absorbent material are useful. Numerous investigations [1–3] have been made to suggest some analysis for the reduction of unwanted noise. Ayub et al. [2] and Huang [3–5] considered the reactive silencer used in HVAC system for reducing ducted tonal fan noise. The duct parallel to x-axis of the inlet/outlet of the expansion chamber was taken to be bounded by membrane with varying height. Because of this variation in height of the membrane the device was tuned which gave stopband for specified frequency. The flexible channels have been discussed by Dowell and Voss [6]. They have analyzed the cavity-backed panel at the low frequency range in the presence of fluid flow. Afterwards Kang and Fuchs [7] has extensively examined their proficiency in the case of cavity-backed micro perforated membrane. Recently Lawrie and Kirby [8,9] investigated the performance of two dimensional modified reactive silencers due to their potential use of hybrid silencers devices in HAVC ducting system. Their investigation proposed that the stopband by the silencer can be broadened and/or shifted with height of the membrane.

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This article focuses on the wave scattering analysis in expansion chamber of two ducts bounded by membrane with different height and a vertical step (flange). The incident mode is scattered into the model spectrum of reflected and transmitted modes. The scattered fields are calculated at the planar junction (discontinuities) by means of the condition of continuity in pressure and normal velocity.

The mode-matching technique is a popular technique to handle the waveguide structures. This technique has been used to analyze the scattered field at the junction. The reflected and transmitted fields are represented in the form of standard Fourier Cosine series. In contrast, the waveguide with higher-order boundary conditions such as the motion along the membrane or thin elastic plate the eigen system becomes non-Sturm–Liouville (SL) and the eigen functions for such systems do not satisfy the standard orthogonality relations (ORs). Firstly this fact was demonstrated by Lawrie and Abrahams [10] where a new form of orthogonality relation (OR) was derived in which the corresponding eigenvalues are the roots of stated dispersion relation for the given non-SL system. The roots for such relations can be found numerically which satisfy the orthogonality proposed in [11]. Later on this technique and related structures has been studied by many authors [12–17] to demonstrate different physical situations.

The boundary value problem is formulated in non-dimensionalized form with respect to length $1/k$ and time $1/\omega$. The fluid velocity potential in two duct regions is expressed in terms of standard Fourier cosine series. As the membrane boundary conditions contain the higher order derivative therefore the piecewise eigen system of the duct region is non-SL. The OR established in [10,11] enables the continuity of pressure and the normal component of velocity to be applied at the mouths of inlet duct. Moreover, the roots of the dispersion relation for the modified silencer are found numerically before the mode-matching equations are truncated and inverted. For a non-dissipative system, such as this, root finding usually presents few problems. It is worthwhile mentioning, however, that the boundary value problem for the model problem falls within the class whereby the mode-matching equations can be recast into root-free form [9] which bypasses the root-finding process. The appropriate physical edge conditions such as zero displacement and zero gradient are applied where membranes join the vertical surface.

In order to validate the problem both mathematically and physically, the low frequency approximation is also made. In most of the cases the low-frequency approximation proves to be an amazingly accurate and a useful implementation in terms of verifying the results obtained via mode-matching approach. However, the low frequency approximation is not formulated as a means of validating the mode-matching solution. The aim is to see how well this approximate method performs against the more accurate mode-matching solution. This could only work for low frequency regime. Another validation of mode-matching approach is confirmed by the accuracy of the modal coefficients by resolving the Gibb's phenomenon [18] and using the Lanczos filter [19,20]. It is worthwhile declaring that this approach successfully handle the issues related to singularity of the velocity field which has recently been discussed by Nawaz and Lawrie [21]. The numerical results are presented to discuss the behavior of scattered coefficients as well as the distribution of power against frequency.

Numerical results also include the case of fundamental as well as second-mode forcing. The fundamental mode mainly carries energy in the wall while the second mode carries energy in the fluid. It is meaningful that the results contrast well for both the incident modes. Along with the continuity conditions, the conservation law of energy which is the fundamental property of the truncated system has also been proved numerically.

The article is sorted in the subsequent order. The two-dimensional problem which incorporates vertical discontinuity is formulated in Section 2. In Section 3, a mode-matching solution is presented using different physical edge conditions as stated earlier while Section 4 is dedicated to produce the low-frequency approximation. The expressions for power distribution are stated in Section 5 whereas few numerical illustrations related to power distribution and modes coefficients are provided in Section 6. The solution to the model problem is validated in Section 7 after which the major findings are concluded in Section 8.

2. Problem statement

Consider a two-dimensional rectangular waveguide containing two duct sections of different heights $\bar{a}$ and $\bar{b}$, where $\bar{a} < \bar{b}$. In dimensional Cartesian frame of reference one section occupying the region at $\bar{x} < 0, 0 < \bar{y} < \bar{a}$ while other at $\bar{x} > 0, 0 < \bar{y} < \bar{b}$. At $\bar{x} = 0$, $\bar{d} < \bar{y} < \bar{b}$, a vertical flange or strip is placed on interface, which joins both the duct sections (see Fig. 1). The lower surface of the waveguide lies along $\bar{y} = 0, -\infty < \bar{x} < \infty$ and is acoustically rigid whilst the upper boundaries are membranes which lie on $\bar{y} = \bar{a}, -\infty < \bar{x} < 0$ and $\bar{y} = \bar{b}, 0 < \bar{x} < \infty$. The sides of the flange are assumed to be rigid at $\bar{x} = 0^-$ and soft at $\bar{x} = 0^+$. A compressible fluid of density ρ and sound speed c occupies the interior region of the waveguide. The harmonic time dependence is taken to be $e^{-i\omega\bar{t}}$, where $\omega = ck$ is the radian frequency in which k is a fluid wave number. The problem is non-dimensionalized with respect to length scale k^{-1} and time scale ω^{-1} under the transformation $x = k\bar{x}$ and $y = k\bar{y}$ etc. The non-dimensional geometry of the problem is shown in Fig. 1.

The non-dimensional velocity potential $\phi(x,y)$ in the waveguide satisfies the Helmholtz's equation

$$\left(\nabla^2 + 1\right)\phi = 0, \tag{1}$$

with unit non-dimensional wave number. The velocity potential $\phi(x,y)$ is defined by

$$\phi(x,y) = \begin{cases} \phi_1(x,y), & x < 0,\ 0 \leqslant y \leqslant a, \\ \phi_2(x,y), & x > 0,\ 0 \leqslant y \leqslant b, \end{cases} \tag{2}$$