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Impact of discontinuous harvesting on fishery dynamics in a stock-effort fishing model ☆

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ABSTRACT

We consider a stock-effort fishing model with discontinuous harvesting strategies. Under some reasonable assumptions on the discontinuous harvesting function, we are able to confirm the well-posedness of the model, describe the structure of possible equilibria as well as establish the stability/instability of the equilibria. Most interestingly, we find that the solutions of the fishing model can arrive at a sliding mode in finite time. A qualitative analysis shows that the goal of maintaining the system at a sustainable equilibrium and optimizing the harvesting can be achieved by introducing the discontinuous harvesting strategies. From the viewpoint of optimal harvesting, we can obtain that discontinuous harvesting strategies are superior to continuous harvesting strategies, which are usually adopted in previous literature. The main difficulty resides in the discontinuity of the model, and is conquered by exploiting the theory of differential equations with discontinuous righthand sides and variable structure system theory.

1. Introduction

Most natural resources are common properties and, when they are exploited, tend to suffer from what is known as the tragedy of the commons [1]. The historical development of many fisheries supports the argument. A particularly spectacular example is the Peruvian anchovy (*Engraulis ringens*) fishery. In an effort to improve the efficient of exploiting stocks and prevent the collapse of key fisheries, a concerted effort was made in the 1950s to develop a quantitative theory of fisheries management, see [2–4]. In the following few decades, the ambitious and productive efforts of biologists to characterize the dynamics of fish populations have been accompanied by a maturation of economists' thinking on the question of efficient exploitation, see [5–11]. The goal of these work is to maintain an optimum sustainable yield and present an optimal harvesting strategies to the fishery management authorities.

There are many theory tools, such as evolutionary game theory [12,13], differential equations theory [14] and statistical theory [15,16], for establishing the theoretical models of populations to study the dynamics of populations. In this study, we mainly adopt the differential equations theory to investigate the dynamics of fishing model. Let us recall the development of mathematical models describing the dynamics of fisheries. The simplest model is introduced by Schaefer [4] as follows:

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$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - qEx, \tag{1}$$

where x denotes the fish populations; E is a positive constant, which represents the fishing effort; r is the intrinsic growth rate of the fish stock; K is the carrying capacity and q is the catchability coefficient. Next, the case that E is variable in time t is considered in [4,7]. Adding an equation of fishing effort to system (1), the following two-dimensional dynamical system is obtained:

$$\begin{cases} \frac{dx}{dt} = rx(1 - \frac{x}{K}) - qEx, \\ \frac{dE}{dt} = kE(pqx - c), \end{cases} \tag{2}$$

where parameters p and c are, respectively, the unit price of the fish and the unit cost of the fishing effort; other variables and parameters have the same meaning as that in model (1). Considering other factors such as age structure, food, predator, the migration of fish and boat in several fishing zones, the variation of the price of the fish and the cost of the fishing effort, model (2) has been generalized and therefore some more complex models have been proposed and studied, see [17–28].

As is well know, when the fish populations are few, the fishery management authorities must close the fishing or decrease the fishing to a very low lever for maintaining a sustainable fishery; while the fish populations are large, the fishery management authorities can raise the fishing to a very high level to increase the economic gains. Therefore, it is reasonable to introduce a harvesting strategies to model (2). The strategies determines the sustainable harvest of the fish in a deterministic setting, when the fish is submitted to invariance effort harvesting. In certain conditions, we can also know the time in which we had better to harvest the fish. Since, for our aim, it is not necessary to regulate the unit of harvesting effort, we can let $q = 1$ to standardize the unit in model (2). The model considered in the paper can be described by the following differential equations with discontinuous right hand sides:

$$\begin{cases} \frac{dx}{dt} = rx(1 - \frac{x}{K}) - \psi(x)Ex, \\ \frac{dE}{dt} = k\psi(x)E(px - c), \end{cases} \tag{3}$$

where $\psi(x)$ is a discontinuous function which denotes the harvesting strategies; Other variables and parameters have the same meaning as that in model (2). To the best of our knowledge, there exist few results on the discontinuous models in ecology, see, for example, [29–32]. In these papers, the theory of variable structure systems [33] as the main tool in the analysis of system dynamics is employed. In this paper, in order to study the dynamics of model (3) more specifically, we not only use the theory of variable structure systems, but also exploit the theory of discontinuous equations with discontinuous righthand sides introduced by Filippov [34].

The paper is organized as follows: In Section 2, some fundamental definitions and tools needed are collected, such as the definition of Filippov solution of model (3) and the chain rule used to differentiate the Lyapunov function along the trajectories of model (3). Section 3 presents the well-posedness, linear stability and global stability of model (3). In Section 4, we firstly present a simple harvesting strategy which is discontinuous. Then, from the viewpoint of optimal harvesting, model (3) is compared with the model under the continuous harvesting strategies. Finally, some conclusions are stated in Section 5.

2. Preliminaries

In this section, we present some definitions and an important lemma, which will be used throughout the paper.

The harvesting strategies in system (3) are modeled with the next class of discontinuous functions.

Definition 1. (Function Class Ψ) We say that $\psi \in \Psi$ if and only if ψ satisfies the following assumptions:

(i) ψ is monotone nondecreasing and has at most a finite number of jump discontinuities in every compact interval;
(ii) $0 \leqslant \psi(x) \leqslant 1$, for any $x \in \mathbb{R}_+$, and $\psi(0) = \psi(0^+) = 0$.

In this paper, $\psi(x)$ can be considered as the proportion of the boats putting in use to the total boats of the fleet. If there is no fish in the fishing zone, the fishing must be stopped. Therefore, $\psi(0) = \psi(0^+) = 0$. The class of discontinuous functions Ψ include a number of harvesting strategies in practice, see Fig. 1.

Since the righthand side of model (3) is a discontinuous function for the fish population x, it is need to specify what is meant by a solution of model (3) with the initial condition. A possible definition, which we shall adopt in the paper, is that of Filippov [34].

Definition 2. A vector function $(x(t), E(t)), t \in I$, is a solution of model (3), with the initial condition $x(t_0) = x_0 \in \mathbb{R}$ and $E(t_0) = E_0 \in \mathbb{R}$, if $(x(t), E(t))$ is absolutely continuous on any subinterval $[t_a, t_b]$ of $I, x(t_0) = x_0, E(t_0) = E_0$, and for almost all (a. a.) $t \in I, (x(t), E(t))$ satisfies the following differential inclusions

$$\begin{cases} \frac{dx}{dt} \in rx(1 - \frac{x}{K}) - \overline{co}[\psi(x)]Ex, \\ \frac{dE}{dt} \in k\overline{co}[\psi(x)]E(px - c), \end{cases} \tag{4}$$

where $\overline{co}[\psi(x)] = [\psi(x^-), \psi(x^+)]$.