Commun Nonlinear Sci Numer Simulat 20 (2015) 614–628

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Bifurcations of nonlinear normal modes via the configuration domain and the time domain shooting methods

Fengxia Wang *

Mechanical and Industrial Engineering, Southern Illinois University Edwardsville, Edwardsville, IL 62026-1805, United States

ARTICLE INFO

Article history:
Received 15 April 2014
Received in revised form 2 June 2014
Accepted 5 June 2014
Available online 17 June 2014

Keywords:
Nonlinear normal modes
Continuation method
Shooting technique
Floquet multiplier

ABSTRACT

This work discusses the two different approaches of applying the shooting method to calculate nonlinear normal modes (NNMs) – the configuration domain integration shooting method and the time domain integration shooting method. The problems and concerns are itemized and analyzed in the work for both of the configuration domain integration and the time domain integration shooting methods. The advantages and disadvantages of these two different approaches of shooting methods are compared. Different algorithms which are able to handle different problems of each approach are developed, respectively. The NNMs are calculated for an autonomous conservative system with inertially coupled quadratic nonlinearities via these two shooting technics. The bifurcations of the NNMs and periodic motions (PMs) are investigated via the combination of the numerical continuation method and the shooting technique. The stability of the NNMs and PMs are determined by the Floquet multiplier.

Published by Elsevier B.V.

1. Introduction

The conception of NNMs is developed by Rosenberg [1,2]. He gave several conditions for the definition of the nonlinear normal modes. But some criterion is too strict, especially the one which requires the mode curve passing through the origin in the n-dimensional configuration space. So his definition should not be the unique definition. In our work, we refer the motions of all masses are equi-periodic and with all the mass simultaneously achieving their maximum displacements as the NNMs of our system. Several general methods used to calculate NNMs have been developed by researchers, including harmonic balance method [3], the method of invariant manifolds as introduced by Shaw and Pierre [4], the method of multiple time scales [5–7] and the asymptotic method [8]. All these methods can give one analytic expression of the normal modes.

Analytical methods are typically only valid for small energy case. Using numerical method, we can get all the NNMs for any fixed energy level and other parameters. Several numerical approaches for the determination of NNMs have been studied by researchers. Among these methods include the algorithm based on pseudo-arc length continuation and the shooting technique developed by Peeters et al. [9–11], variations of the pseudo-arc length continuation algorithm [12], numerical time integration algorithm with a cost function seeking the initial condition of the periodic solution [13], as well as the numerical method using a global solution technique [14]. Among all these different methodologies, the shooting method is an efficient method to calculate NNMs for both high energy and strong nonlinear systems [15]. In the work [15], the

* Tel.: +1 6186502540.
E-mail address: fwang@siue.edu

http://dx.doi.org/10.1016/j.cnsns.2014.06.008
1007-5704/Published by Elsevier B.V.

configuration integration shooting method is employed by applying the least-action principle. The application of the least-action principle treats one degree of freedom as independent variables and all the other degrees of freedom are single value function of the independent variables. This requirement restricted the convergence of the NNMs computation as the system parameter changes.

Therefore, it is necessary to develop a solution approach based on shooting technique which is capable of calculating NNMs via the time domain integrations. The present study is an effort in this direction, and details related to time domain integration "shooting" to obtain the NNMs are described in this work.

Shooting method is developed for solving very general two-point boundary-value problems [16,17], by assuming the missing initial conditions and integrating the differential equations forward over the interval. The success of the shooting method requires two essential conditions: (1) the initial value problem has a solution over the integration interval; (2) for assumed initial conditions near the correct initial conditions, its solution must be close to the true solution. Problems satisfy these two properties are said to be well-posed, well-conditioned, or stable. On the other hand problems cannot satisfy these properties are said to be not well-posed, ill-conditioned, or unstable. The ill-condition or unstable problem is inherent in the problem and is quite independent of the numerical method used to integrate the differential equations. A representative ill-condition or unstable example is given by Troesch B. A. [18] and Keller [16]: $\ddot{y} = 16\sin 16y$ with the boundary conditions $y(0) = y(1) = 0$. The unique solution of this problem indicates the true missing initial condition is $\dot{y}(0) = 0$. If the problem is solved by shooting method with guessed initial conditions $y(0) = 0,\ \dot{y}(0) = \varepsilon$, and $|\varepsilon| > 1e-7$, the solution with the guessed initial condition is singular in the integration interval [0,1]. Techniques have been developed to deal with such kind of ill-condition or unstable problems, including shooting method with orthogonalization [19,20], quasi-linearization [21], finite difference method [22], and parallel shooting method [16,23]. However these techniques also introduce expensive computation.

As to the calculation of the NNMs by the shooting method, in our work a spring-mass-pendulum example is used to demonstrate the difference between the time domain integration and the configuration domain integration shooting techniques. For this particular example, the time domain integration shooting technique suffers from the singularity problem for some specific NNM with a vanishing degree of freedom, while the configuration domain integration shooting technique does not have such kind of problem. However, as we mentioned before, the configuration integration shooting method fails to converge for the calculating of NNMs, in which all the other degrees of freedom cannot be represented as a single value functions of the independent variable.

Note that the stability of the calculated NNMs via the time integration shooting technique does not determine the ill-condition or well-condition of the problem. Or in another word, if the calculated NNM is unstable, it does not mean this is an ill-condition or unstable shooting problem in time integration. If only if the unstable NNM "blows up" in the integration time interval (typically is one period), the ill-condition or unstable problem is struck in the shooting process, and the true unstable NNM will not be able found with satisfied accuracy. Most time, even the NNM is unstable, in the integration time interval it will not blow up. However, if the unstable NNM goes away from the true solution with exponential level growth, then the ill-condition or unstable shooting problem is encountered. For the time integration shooting technique, even if the true solution of NNM is stable, because of the vanish of some degrees of freedom in the NNM, singularity issues may triggered in the Newton–Raphson algorithm which is commonly used to find the adjustments of the missing initial conditions of the integration. For configuration integration shooting considering the integration is with regards to a configuration independent variable instead of time variable, as long as a proper independent variable is selected, the existence of the nonlinear normal mode in the limited configuration space guarantees the ill-condition or unstable problem will not happen.

2. Calculating NNMs by the configuration integration shooting method

Suppose that we have a n degrees-of-freedom discrete model, the discretized governing equations of motion with nonlinearities can be written as,

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{K}\mathbf{y} + \mathrm{N}(\mathbf{y}, \dot{\mathbf{y}}, \ddot{\mathbf{y}}) = 0, \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are mass and constant stiffness matrices; $\mathbf{y}, \dot{\mathbf{y}}$, and $\ddot{\mathbf{y}}$ are n dimensional displacement, velocity and acceleration vectors.

2.1. Boundary value problem construction in configuration space

In order to get the nonlinear normal mode trajectories in the configuration space [15], we apply the principle of least action to Eq. (1). For a nonlinear normal mode at the simplest level, all but one of the variables or coordinates $y_i, i = 2, 3, \ldots$ can be expressed as single-valued functions of the coordinate y_1 or any specific coordinate y_c. Here we use y_1 as the independent variable to illustrate the calculation process. To simplify the expression in the following analysis, variable substitutions are introduced,

$$\mathbf{z}_d = [z_1, z_2, \ldots, z_{n-1}]^{\mathrm{T}} = [y_2, y_3, \ldots, y_n]^{\mathrm{T}}, \tag{2}$$