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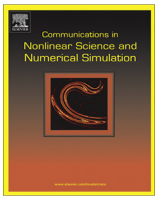

Period doubling cascades of prey–predator model with nonlinear harvesting and control of over exploitation through taxation

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ABSTRACT

The present study investigates a prey predator type model for conservation of ecological resources through taxation with nonlinear harvesting. The model uses the harvesting function as proposed by Agnew (1979) [1] which accounts for the handling time of the catch and also the competition between standard vessels being utilized for harvesting of resources. In this paper we consider a three dimensional dynamic effort prey–predator model with Holling type-II functional response. The conditions for uniform persistence of the model have been derived. The existence and stability of bifurcating periodic solution through Hopf bifurcation have been examined for a particular set of parameter value. Using numerical examples it is shown that the system admits periodic, quasi-periodic and chaotic solutions. It is observed that the system exhibits periodic doubling route to chaos with respect to tax. Many forms of complexities such as chaotic bands (including periodic windows, period-doubling bifurcations, period-halving bifurcations and attractor crisis) and chaotic attractors have been observed. Sensitivity analysis is carried out and it is observed that the solutions are highly dependent to the initial conditions. Pontryagin's Maximum Principle has been used to obtain optimal tax policy to maximize the monetary social benefit as well as conservation of the ecosystem.

1. Introduction

The classical ecological models of interacting populations have been investigated extensively in the literature. In recent years the growing demand for more food/ resources has resulted in over exploitation of several biological resources. Therefore there is a need for a sustainable development policy in various spheres of human activity to protect ecosystems. In particular bioeconomic modeling is concerned with scientific management of the exploitation of renewable resources like fisheries and forestry. Hence harvesting of ecosystems has been of interest to economists and ecologists for some time now. A harvesting policy refers to the management of biological resources by systematically controlling the period, intensity and type of harvesting. The primary objective here is to maximize productivity without depleting or driving the stocks to extinction. In a harvesting model with two species (prey and predator) harvesting of the prey species is of interest. It is generally not possible to control the consumption of the prey by the predator species, but control may be exercised by removal (harvesting) of a certain quantity of the prey species. Most of the studies in this direction have used Gordon-Schaefer mathematical model or its variants to study harvesting [5–7,18]. It may be pointed out that the Gordon-Schaefer model considers a logistic growth term for the ecological resources.

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Harvesting generally has a strong impact on the dynamics of biological resources. The severity of the impact, which depends on the nature of the implemented harvesting strategy, may range from rapid depletion to complete preservation of a population. Basically three types of harvesting are reported in literature (i) constant harvesting where a constant number of individuals are harvested per unit of time, (ii) proportional harvesting $h(x) = qEx$ that means the number of individuals harvested per unit of time is proportional to the current population x (here q is the catchability coefficient, E is the effort applied to harvest individuals which is measured in terms of number of (standard) vessels being used to harvest the individual population) and (iii) nonlinear harvesting $H(x) = qEx/(m_1E + m_2x)$ (Holling type-II), where m_1, m_2 are suitable positive constants. Many researchers have analyzed mathematical models using Holling type-II harvesting by considering different type of growth depending upon the species and their interactions [12,20,31] etc. Agnew [1] proposed a different type of harvesting function for logistically growing single species. It accounts for the handling or processing time of the catch as well as for the competition between standard vessels which are utilized for harvesting of resources.

In general the studies of ecological models are devoted to the existence, stability and bifurcation of equilibria and persistence of solution. Deterministic chaos has been an active area of research in various branches of sciences. It has also become a subject of intense research in ecology ever since May [24,25] demonstrated that simple models can lead to complex dynamics. The ecological systems contain the ingredients for possible occurrence of chaos. Recently it has been seen that the chaotic dynamic may play an important role in continuous time models for ecological systems. For instance, there are evidences that the real time evolution of the species involved in food-chains could be characterized by chaotic attractors. Hastings and Powell [14] proposed three species food chain system which is a coupled system of nonlinear equations. They numerically simulated the behavior of the model and established the occurrence of chaos for biologically realistic parametric values. Hasting et al. [15] pointed out that sensitive dependence on initial conditions is the best heuristic definition of dynamical chaos. There are several other papers for food chain model where appearance of chaos in the system has been observed [2,4,26,32] etc. Complex dynamics in a prey–predator system with multiple delays has been discussed by Gakkhar and Singh [11]. Existence of chaos in two-prey and one-predator system is observed in [19]. A three-species ecological model with impulsive control strategy has been proposed by Yu et al. [34].

The period-doubling route to chaos has attracted many researchers and it is an object of great interest for different complex phenomena observed in nonlinear dynamical systems. The periodic-doubling route to chaos has been observed in many physical, chemical and ecological models when they change from simple periodic to complex aperiodic motion. The research of last two decades demonstrates that very complex dynamics (e.g. quasi-periodic or even chaos) can arise in continuous time ecological models with three or more species. Much attention has been paid to study the transition from periodic to chaotic dynamics. Rai and Sreenivasan [29] considered a food chain involving three species to show the presence of a period-doubling scenario leading to chaos. Lacitignola et al. [22] pointed out a period-doubling route to chaos in time-dependent regimes of a tourism-based socialecological system. Their analysis showed that the dynamical system may in fact evolve toward an aperiodical dynamical state in many ways, thus resulting in different scenarios for the transition to chaos. He and Lai [16] analyzed a period-doubling route to chaos in discrete-time predator–prey system. Period doubling cascades in a predator–prey model with a scavenger has been observed by Previte and Hoffman [28].

2. Mathematical model

In this section we develop a prey predator type model for conservation of ecological resources through taxation with nonlinear harvesting and we provide the result for positivity and uniform boundedness of solutions of the proposed model.

2.1. General mathematical model

The most general prey-dependent continuous time model describing the dynamics of prey–predator population based on the fact that functional response over ecological time scale depends on the density of prey only can be represented by [9]

$$\begin{cases} \frac{dx}{dt} = xg(x) - yp(x), \\ \frac{dy}{dt} = y(-d + \theta p(x)), \end{cases} \tag{1}$$

subject to positive initial conditions

$$x(0) > 0, y(0) > 0, \tag{2}$$

where $x(t)$ and $y(t)$ are the densities of prey and predator populations at time t. $g(x)$ is the per capita net prey growth in absence of predator, $p(x)$ represents the functional response of predator, θ is the conversion rate and d is the natural death rate of the predator. It is assumed that the functions $g(x)$ and $p(x)$ are sufficiently smooth so that the existence and uniqueness of solution for all positive t are satisfied for initial value problem 1,2. In addition, it is assumed that

(i) $g(0) = 0$ and there exists $k > 0$ such that $g(x) > 0$ on $0 < x < k$, $g(k) = 0$ and $g(x) < 0$ for $x > k$,
(ii) $p(0) = 0$ and $p'(x) \neq 0$.