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Sampled-data nonlinear observer design for chaos synchronization: A Lyapunov-based approach

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ABSTRACT

This paper considers sampled-data based chaos synchronization using observers in the presence of measurement noise for a large class of chaotic systems. We study discretized model of chaotic systems which are perturbed by white noise and employ Lyapunov-like theorems to come up with a simple yet effective observer design. For the choice of observer gain, a suboptimal criterion is obtained in terms of LMI. We present semiglobal as well as global results. The proposed scheme can also be extended for discrete-time chaotic systems. Numerical simulations have been carried out to verify the effectiveness of theoretical results.

1. Introduction

The concept of chaos synchronization which is introduced by Pecora and Carroll [1] is received wide attention over the past decades because of its broad range of applications, including secure communication [2], chemical reactions [3], information processing [4], etc. This phenomenon occurs when two chaotic oscillators are coupled, or when a chaotic oscillator drives another chaotic oscillator.

The methods used to realize chaos synchronization can be broadly classified into two categories. In the first one, synchronization is performed in master–slave structure. That is, a controller is employed such that the behavior of slave mimics that of master; see for example [5–8]. The second category refers to synchronization of two chaotic systems with different initial conditions and/or parameters which yields simpler synchronization schemes and can be regarded as an observer design problem; see for example [9–12]. While the theory of these approaches is mainly developed in continuous-time, it is important to study them in discrete-time domain because most implementation methods are based on sampled-data and digital computers.

Based on the above motivation, we intend to design a simple observer-based chaos synchronization scheme in discrete-time domain. Thus, this paper belongs to the second category and deals specifically with sampled-data based chaos synchronization. Faced with design in discrete-time framework, there are two different approaches. In the first one, design is performed in continuous-time, and then, discretized and implemented using sample/hold devices [13]. In this method, it is vital to show performance recovery of continue-time design under sampled-data implementation. The second approach uses

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an approximate discretized model of the plant and the tools in discrete-time control theory [14]. In this paper, our interest is to employ the second method because this allows us to generalize the synchronization scheme to discrete-time chaotic systems such as logistic map [15] and Henon map [16]. This generalization is based on the fact that all the design steps are based on difference inclusions rather than differential equations.

From practical point of view it is important to consider the effects of measurement noise which is a considerable source of disturbance. Thus we consider the case that chaotic states are perturbed by white noise. We adopt the particular structure of the observer from [17] and employ the Lipschitz-like conditions similar to [18] in order to describe nonlinearities of the chaotic system. Then rigorous stability analysis are carried out based on Lyapunov-like theorems for stability of discrete-time stochastic systems and an LMI-based suboptimal criterion is derived for the observer gain in order to optimize the bound on estimation error. The proposed observer has the advantage that the attraction basin of the equilibrium point (i.e. the origin) can be arbitrarily large. In fact, most of the conventional state estimators such as extended Kalman filter (EKF) [19] are based on linearized equations and do not guarantee the convergence in global sense for nonlinear systems. If the initial state of the system lies outside the attraction basin, the state estimation error will not converge to the origin. However, by virtue of Lyapunov-like theorems and some mathematical manipulations that we have utilized here, our result will hold semiglobally (not locally).

While recent work on observer-based chaos synchronization focused on noise compensation and global convergence, there is little wok on providing systematic approaches to optimize the estimation error; see for example [20–32]. The major contribution of this paper is to introduce an LMI-based suboptimal criterion for the choice of observer gain to optimize the bounds on estimation error which can improve the estimation performance. In summary the features of our work include:

- General design framework for both continuous-time and discrete-time chaotic systems.
- Suboptimal criterion for the choice of observer gain to optimize the estimation in the presence of measurement noise.
- Semiglobal convergence results.

These features can be rarely found all together in the previous work reported in the literature.

This paper is organized as follows: In Section 2, problem formulation is given. Main results are presented in Section 3. An illustrative example and simulation studies are included in Section 4 and finally, we conclude the paper in Section 5.

Notation. Throughout the paper, bold face lower-case letters indicate vectors and the upper-case, matrices. We also use the notation $\mathcal{E}[\cdot]$ to denote expectation value.

2. Problem formulation

We consider an n-dimensional nonlinear difference equation as follows

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k), \tag{1}$$

where $\mathbf{x} \in \mathcal{R}^{n\times 1}$ is called state vector and $\mathbf{f}(\cdot)$ is a nonlinear deterministic vector in difference equations. Many chaotic systems can be represented in the form of (1). Consider, for example, the Lorenz equations [33] represented by

$$\begin{cases} \dot{x}_1 = \sigma(x_2 - x_1), \\ \dot{x}_2 = rx_1 - x_2 - x_1x_3, \\ \dot{x}_3 = x_1x_2 - bx_3. \end{cases} \tag{2}$$

Using the Euler approximation method, the system (2) is discretized as follows

$$\begin{pmatrix} x_{k+1}^1 \\ x_{k+1}^2 \\ x_{k+1}^3 \end{pmatrix} = \begin{pmatrix} (1 - T\sigma)x_k^1 + T\sigma x_k^2 \\ Trx_k^1 + (1 - T)x_k^2 - Tx_k^1x_k^3 \\ (1 - Tb)x_k^3 + Tx_k^1x_k^2 \end{pmatrix}, \tag{3}$$

where T is the step size. Clearly, Eq. (3) fits the nonlinear difference equation (1). Similarly, other continuous-time chaotic systems can be discretized and transformed to this form. It is worth noticing that chaotic maps such as logistic map which is described by

$$x_{k+1} = \mu x_k(1 - x_k) \tag{4}$$

are also in the form of (1). Thus, Eq. (1) refers to a large class of chaotic systems and the results presented here are applicable to them. We consider the case that the measured chaotic states are perturbed by white noise. For this purpose, consider the following output equation

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{D}\mathbf{v}_k, \tag{5}$$

where $\mathbf{y} \in \mathcal{R}^{m\times 1}$ is the output vector, $\mathbf{H}$ and $\mathbf{D}$ are constant matrices with appropriate dimensions and $\mathbf{v}$ denotes white noise with zero mean and covariance indicated by n_v. The problem is to design a state observer to estimate the state $\mathbf{x}$ from the perturbed observations $\mathbf{y}$. We employ the following observer