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Stagnation point flow and heat transfer over a non-linearly moving flat plate in a parallel free stream with slip

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ABSTRACT

An analysis is presented for the steady boundary layer flow and heat transfer of a viscous and incompressible fluid in the stagnation point towards a non-linearly moving flat plate in a parallel free stream with a partial slip velocity. The governing partial differential equations are converted into nonlinear ordinary differential equations by a similarity transformation, which are then solved numerically using the function bvp4c from Matlab for different values of the governing parameters. Dual (upper and lower branch) solutions are found to exist for certain parameters. Particular attention is given to deriving numerical results for the critical/turning points which determine the range of existence of the dual solutions. A stability analysis has been also performed to show that the upper branch solutions are stable and physically realizable, while the lower branch solutions are not stable and, therefore, not physically possible.

1. Introduction

The boundary layer flow due to a continuously moving solid surface has received considerable attention since the pioneering study by Sakiadis [1,2]. The principal reason for interest in this problem is its practical relevance in various manufacturing processes in industry such as the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the boundary layer along a liquid film in condensation processes. Apart from this, the mathematical model considered in this context has significance in studying several problems of engineering, meteorology, oceanography, design of supersonic and hypersonic flights, etc. Sakiadis [1,2] showed that the partial differential equation of motion governing the steady and laminar flow caused by a continuously moving solid plate can be reduced by a similarity transformation to an ordinary differential equation. This equation is identical to the momentum equation for the boundary layer flow that develops between a constant fluid stream and a stationary flat plate first considered by Blasius [3]. However, the boundary conditions are different and so is the resulting velocity profile. Unlike Blasius [3] flow, the continuous moving surface sucks the ambient fluid and pumps it again in the downstream direction. Dual solutions were found when the plate advances toward the oncoming stream.

The boundary layer problem due to Sakiadis [1,2] has been extended in a variety of ways during the subsequent decades. Klemp and Acrivos [4,5] studied the motion induced by impermeable finite and semi-infinite flat plates moving at constant velocity beneath a uniform mainstream. Later, Hussaini and Lakin [6] showed that the solutions for such boundary layer problems exist only up to a certain critical value of a moving parameter. Further, Hussaini et al. [7] considered the problem studied by Klemp and Acrivos [4] with a view to obtaining analyticity of the solutions. Riley and Weidman [8] analyzed the

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nature of the lower branch dual solution as the shear stress (skin friction) and plate velocity simultaneously tend to zero, showing that the boundary layer lifts off the plate, even in the absence of blowing. Vajravelu and Mohapatra [9] studied the problem considered by Hussaini et al. [7] under the effect of injection. They showed that there exists a range of values of the moving and injection parameters for which the governing ordinary differential equation admits analytic solutions. The simultaneous effects of normal transpiration through and tangential movement of a semi-infinite plate on self-similar boundary layer flow beneath a uniform free stream has been studied by Weidman et al. [10]. The stability analysis showed that, for each value of the transpiration parameter, upper solution branches are stable, while lower solution branches are unstable.

The thermal behavior for a continuous moving surface was first examined by Siekman [11]. Erickson et al. [12] examined this problem through an approximate integral analysis. Using a similarity transformation with a numerical approach, Tsou et al. [13] studied the heat transfer behavior at several values of the Prandtl number. Takhar et al. [14] were apparently the first who have extended Sakiadis [2] problem to include effects of temperature-dependent physical fluid properties. Pop et al. [15] considered only the effect of an inversely linear variation of the viscosity with temperature for two distinct Prandtl numbers of 0.7 and 10. Jacobi [16] proposed a correlation equation with applicability for all Prandtl numbers, one that properly reflects the physics of the moving surface. The development of the momentum and thermal boundary layers over a semi-infinite flat plate has been studied by Kumari and Nath [17] when the external stream, as well as the plate are impulsively moved with constant velocities. Further, Andersson and Aarseth [18] considered the Sakiadis [2] problem with variable fluid properties. The case with only variable viscosity was recovered as a special case for which a Blasius-type transformation applies. New numerical results for water at atmospheric pressure were reported. Abdulhafez [19], and Chappidi and Gunnerson [20] analyzed two sets of boundary value problems for the moving plate in two separately cases: $U_w > U_\infty$ or $U_\infty > U_w$, where U_w is the constant plate velocity and U_∞ is the constant free stream velocity, respectively. Later, Afzal et al. [21] and Afzal [22,23] formulated the problem of the moving flat plate by introducing a single set of equations and using the composite reference velocity $U(= U_w + U_\infty)$ with a constant moving parameter $r = U_\infty/U$, where $0 \leqslant r \leqslant 1$. It is worth mentioning that for $r = 1$ ($U_w = 0$), the problem reduces to the classical Blasius [3] flat plate problem, a detailed numerical solution of this problem being presented by Cortell [24]. On the other hand, for $r = 0$ ($U_\infty = 0$) the problem has been solved by Afzal [22,23]. It has been shown in these papers that for $1 < r < 1.548$ dual solutions of the governing similarity (ordinary differential) equations exist and no solution exists for $r > 1.548$. Other important papers on parallel and reverse flows with heat transfer are those by Lin and Huang [25], Sparrow and Abraham [26], and Abraham and Sparrow [27]. Cortell [28] has reported numerical results for the flow and heat transfer past a parallel moving sheet based on the moving parameter r in the range $0 \leqslant r \leqslant 1$. Finally, we mention the papers on the boundary layer flow and heat transfer with variable fluid properties on a moving flat plate in a parallel free stream [29] and the MHD boundary layer flow due to a moving wedge in a parallel stream with induced magnetic field [30], respectively. It should be also mentioned the excellent review paper by Wang [31], where the advantages of the similarity solutions of a viscous fluid due to a stretching boundary were shown.

The aim of the present paper is to study theoretically the stagnation point flow and heat transfer over a non-linearly moving flat plate in a parallel free stream with a partial slip velocity. Effects of slip conditions are very important for fluids that exhibit wall slip such as emulsions, suspensions, foams, polymer solutions, etc. Fluids exhibiting slip are important in several technological applications such as in the polishing of artificial heart valves and internal cavities [32]. Navier [33] proposed a slip boundary condition wherein the slip depends linearly on the shear stress (or skin friction). A number of models have been used for describing the slip that occurs at solid surfaces. A large number of authors such as Andersson [34], Wang [35,36], Hayat et al. [37], Zhu et al. [38], Fang et al. [39–42], etc. We mention also to this end that Rohni et al. [43] have studied in a recent paper, in completing the results reported by Cortell [44], the effects of suction/blowing and viscous dissipation on the steady non-linear viscous flow and heat transfer over a horizontal shrinking permeable surface of variable temperature or heat flux with a power-law velocity. Particular attention has been given to deriving numerical results for the critical/turning points which determine the range of existence of multiple solutions. Dual and triple solutions are found to exist for certain parameters.

2. Basic equations for the flow

Consider the steady two-dimensional flow and heat transfer of a viscous incompressible fluid near a stagnation point past a moving surface coinciding with the plane $y = 0$. The plate is moving into or out of the origin with the velocity $u_w(x) = cx^n$ in an outer (inviscid) flow of the velocity $u_e(x) = ax^n$, where a, c and n are positive constants. Here x and y are the Cartesian coordinates measured along the plate and normal to it, respectively. The fluid occupies the upper half plane ($y > 0$). It is also assumed that the temperature of the plate is $T_w(x)$, while the constant temperature of the ambient fluid is T_∞. Under these assumptions, the unsteady governing continuity, momentum and energy boundary layer equations are [45]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \quad (2)$$