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A new sliding control strategy for nonlinear system solved by the Lie-group differential algebraic equation method

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ABSTRACT

For a control problem of nonlinear system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u, t)$, the optimal control by minimizing a performance index is reformulated to be a set of differential algebraic equations (DAEs) with the Lagrangian being partially replaced by an exponentially time-decaying constraint: $L_1(\mathbf{x}, t) = A_0 e^{-\alpha t}$, and meanwhile the control force is bounded by $|u| \leqslant u_{\max}$. Then, we develop an implicit $GL(n, \mathbb{R})$ Lie-group DAE (LGDAE) method to find $u(t)$ by solving the DAEs: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u, t)$ and $L_1(\mathbf{x}, t) - A_0 e^{-\alpha t} = 0$. Similarly, we propose a new sliding mode control (SMC) strategy by using the LGDAE to solve the control force, where in addition to the equivalent control force we add a compensated control force which is used to quickly steer and continuously enforce the state trajectory on the sliding surface. This novel SMC is robust and is chattering-free for regulator problem and finite-time tracking problem of nonlinear systems. Furthermore, we combine the above two methods as being a two-stage controller for the forced nonlinear Duffing oscillator by stabilizing it to an equilibrium point. The present SMC together with the LGDAE is also used to stabilize the state trajectory of some uncertain chaotic systems to a desired state point. Its robustness against uncertainty is obvious.

1. Introduction

Sometimes we may encounter the problem that some external forces are not yet known, but service for a specific purpose of controlling the nonlinear plant to a desired goal. Then the resulting problem is a control problem, of which we need to design a suitable controller to achieve the desired goal. In the class of optimal control problems, the control forces are intentionally designed such that a specified cost functional which weights the cost of control versus the allowed response is minimized. The control of nonlinear structural systems has gained much attention in the past several decades, and different controllers were proposed for the applications to different disciplines [1,2]. In the realm of nonlinear structural control, Davies [3] has studied the time optimal control problem of the Duffing oscillator. Van Dooren and Vlassenbroeck [4], and Vlassenbroeck and Van Dooren [5] have introduced a direct method by the Chebyshev series expansion method to solve the control problem of the Duffing oscillator [6,7]. Razzaghi and Elnagar [8] have applied a pseudo-spectral method and Lakestani et al. [9] have applied a semi-orthogonal spline wavelets to solve this problem. As a result, all the above methods required to solve a rather complicated system of nonlinear algebraic equations. Attempting to overcome these difficulties, Liu [10] has presented an alternative approach based on the Lie-group adaptive method, where the governing equation of nonlinear system is viewed to be the major part and the performance index as being a subsidiary target equation to be minimized.

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Besides the conventional performancen-index based control theory, Utkin [11–13] has developed a sliding mode control (SMC) theory, which is also known as a variable structure control, and which provides a powerful and robust control mechanism for linear and nonlinear systems [11,14–17]. In its earlier approach, an infinite frequency control switching is required to maintain the trajectories on a prescribed sliding surface and then eventually to enforce the orbit tending to the equilibrium point along the sliding surface. However, in practice the system states not really locate on the designed sliding surface after reaching it due to numerically discretizing errors, signal noises as well as structural uncertainties in the dynamical equations. Since the controller was fast switched during operation, the system undergone an oscillation crossing the sliding plane. Around the sliding surface is often irritated by high frequency and small amplitude oscillations known as chattering [18]. The phenomenon of chattering is a major drawback of SMC, which makes the control power unnecessarily large. To eliminate the chattering, there were some methods being developed [18–21].

The SMC is widely used as a powerful method to tackle uncertain nonlinear systems [22–24]. Roopaei et al. [25] have used an adaptive gain fuzzy SMC to control nonlinear chaotic systems in the presence of model uncertainty and external disturbance. Around the sliding surface, it appears chattering, which is undesirable because it involves high control activity and may excite high frequency dynamics which is neglected in the modelling course. In this paper we propose a simple sliding mode control strategy by adding an auxiliary controller which is obtained by solving a set of differential algebraic equations (DAEs). The resultant controlled system is chattering-free.

This paper is arranged as follows. The preliminaries of conventional performance-index based control law are briefly sketched in Section 2. Then we introduce a novel approach to replace the optimal control problem of nonlinear system in Section 3 by directly specifying a time-decaying Lagrangian function, such that we can transform the optimal control problem into a system of differential algebraic equations (DAEs). Then we derive a simple $GL(n,\mathbb{R})$ Lie-group integration method for the system of nonlinear ODEs. Taking advantage of the Lie-group property, we propose an implicit integration technique in Section 4, such that we can derive a simple Newton iterative scheme to compute the control force in Section 5. In the numerical experiments and examples of dynamical systems to be tested we include linear systems and nonlinear systems, and the latter are further classified as deterministic systems and uncertain systems. The chaotic examples of Duffing oscillator, Lorenz system, Liu system and Lorenz–Stenflo system are included. The examples of regulator problems are given in Section 6 by applying the Lie-group DAE (LGDAE) method to solve them. In Section 7 we modify the conventional sliding mode control (SMC) method and apply the Lie-group DAE (LGDAE) method to find the control force. In Section 8 we give some deterministic systems where the Fuller problem, some numerical examples of regulator problem and finite-time tracking problem of chaotic Duffing system are examined to test the performance of the newly developed SMC strategy and the LGDAE method. In Section 9 we use the new SMC and the LGDAE to solve the stabilization problems of some uncertain chaotic systems. Finally, we draw some conclusions in Section 10.

2. Nonlinear plant and conventional control law

The linear quadratic (LQ) optimal control methodologies provide a complete multi-variable design and synthesis theory. However, the conventional theory gives only an optimal control law for the linear plant without considering the external disturbance, *i.e.*,

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad \forall t \in [0, t_f], \ \mathbf{x}(0) = \mathbf{x}_0, \tag{1}$$

where $[0, t_f]$ is a time interval during which the plant is under a control force $\mathbf{u}(t)$.

Upon minimizing the following performance index:

$$J = \int_0^{t_f} [\mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^{\mathrm{T}}(t)\mathbf{R}\mathbf{u}(t)]dt, \tag{2}$$

where $\mathbf{Q} \in \mathbb{R}^{n\times n}$ and $\mathbf{R} \in \mathbb{R}^{u\times u}$ are positive semi-definite and positive definite, respectively, and the superscript T stands for the transpose, the optimal control law is found to be

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{R}_x(t)\mathbf{x}(t) \quad \forall t \in [0, t_f], \tag{3}$$

where $\mathbf{R}_x$ is a Riccati matrix obtained by solving the following Riccati differential equation:

$$\dot{\mathbf{R}}_x + \mathbf{Q} + \mathbf{R}_x\mathbf{A} + \mathbf{A}^{\mathrm{T}}\mathbf{R}_x - \mathbf{R}_x\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{R}_x = \mathbf{0}, \quad \mathbf{R}_x(t_f) = \mathbf{0}. \tag{4}$$

Eq. (3) presents a state feedback control law. Unfortunately, the control law (3) upon applied to an externally excited plant is not the optimal one.

In the conventional state feedback control theory, Eq. (4) is solved numerically backward in time, and with normal values of weighting matrices and structural properties, the Riccati matrix $\mathbf{R}_x(t)$ remains constant almost over the entire time duration $[0, t_f]$ except that very near the terminal time t_f; hence, we usually set $\mathbf{R}_x$ to be a constant matrix satisfying

$$\mathbf{Q} + \mathbf{R}_x\mathbf{A} + \mathbf{A}^{\mathrm{T}}\mathbf{R}_x - \mathbf{R}_x\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{R}_x = \mathbf{0}. \tag{5}$$

For its wide application in control theory, there are many techniques to solve the above algebraic Riccati equation [26].

The purpose of this paper is to compute a single control force u in the following nonlinear system: