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Synchronization of chaotic Lur'e systems with quantized sampled-data controller ☆

Xiaoqing Xiao [a,b], Lei Zhou [b,*], Zhenjuan Zhang [b]

[a] School of Automation, Nanjing University of Science and Technology, Jiangsu 210094, PR China
[b] School of Electronics and Information, Nantong University, Jiangsu 226019, PR China

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A B S T R A C T

In this paper, we consider the master–slave synchronization problem of chaotic Lur'e systems. It is assumed that only quantized sampled measurements are available for the controller. By modeling the synchronization error system as an input-delay system and constructing a new Lyapunov functional, a new sufficient condition and feedback controller design method for global exponential asymptotical synchronization of master and slave system are obtained. The proposed approach has taken the feature of sample-induced delay into consideration and simulation results show the less conservativeness.

1. Introduction

Since the pioneering work of Pecora and Carroll [1], synchronization of chaotic systems has received considerable attentions due to its potential applications in secure communication, chaos generator design, biological systems, and many other fields. It is well known that Chua circuit [2], coupled Chua circuits [3], n-scroll circuits [4], and other chaotic and hyperchaotic systems can be represented in Lur'e form in which the nonlinearity satisfies a sector condition [5]. Over the last decade, several synchronization schemes for chaotic Lur'e systems have been proposed [6–8].

On the other hand, with the development of computer technology, microelectronics, and communication networks, discrete-time controllers are also proposed in many practical applications, which only need the sampled-data of the measurements of the systems at discrete time instants. So far two main approaches have been proposed for the sampled-data robust stabilization problem. The first one is based on the lifting technique [9], in which the problem is transformed to the equivalent finite-dimensional discrete problem.

The second approach is to represent the closed system as a hybrid system which combines both continuous and discrete-time signals. Modeling of continuous-time systems with digital control as continuous systems with delayed control input was introduced by [10]. Recently, this approach was applied to robust sampled-data stabilization [11]. In [12], a new input delay approach is proposed to the robust sampled-data stabilization problem. The sampled-data stabilization is addressed by solving the problem for a continuous-time system with uncertain but bounded time-varying delay in the control input.

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* Corresponding author. Tel.: +86 051385012626.
E-mail address: zlxxq80@ntu.edu.cn (L. Zhou).

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More recently, the sampled-data stabilization approach has been applied to the synchronization of chaotic systems [13–15]. More specially, in [14], based on the representation of the sampled-data error system as a time-varying delay system [12], sufficient conditions for global asymptotic synchronization of chaotic Lur'e systems were obtained using the free-weighting matrix approach, and the maximum sampling interval h guaranteeing the global asymptotical synchronization expressed in terms of linear matrix inequalities. Further improvements have been made in [15,16] such that the synchronization sampled-data controllers allow for a bigger upper bound of sampling interval.

It should be mentioned that the synchronization schemes proposed in [13–15] are based on the standard assumption that there is a perfect communication channel between the measurement of the system and the input of the controller. However, we note that in a network environment, sampled signals are usually quantized before being transmitted. The quantizer can be regarded as a coder which converts the continuous signal into piecewise continuous signal taking values in a finite set [17]. For the synchronization problem, an immediate concern arising from the introduction of the quantizer is the quantization effect, which is closely related to the capacity required to transmit the control signals. However, to the best of our knowledge, the study of synchronization of chaotic systems with quantized sampled-data controller has not been reported for chaotic Lur'e systems in the literature, which motivates the present study.

In this letter, we investigate the synchronization problem for chaotic Lur'e systems with quantized sampled-data controller. The measurements of the master system is sampled and then quantized. The sampling behavior is dealt with via a input delay system approach, and the measurement quantization is treated using a sector bound method. By using of Lyapunov functional method, the synchronization problem with quantized sampled-data controller is first solved for chaotic Lur'e systems, and a new controller design method is obtained. We mention that our approach makes full use of the information about the sample-induced delay and the derived result is expected to be less conservative compared with the existing results, which is illustrated by numerical simulations.

Notation: Throughout this paper, $\mathbf{R}^n$ denotes the n-dimensional Euclidean space. I is the appropriately dimensioned identity matrix, W^T denotes transpose of matrix W, $W > 0$ means that W is positive definite. Asterisk '$*$' in a symmetric matrix denotes the entry implied by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation

In this letter, we consider the following master–slave synchronization scheme:

$$\mathcal{M}: \begin{cases} \dot{x}(t) = Ax(t) + Bf(Dx(t)), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

$$\mathcal{S}: \begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bf(D\hat{x}(t)) + u(t), \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{2}$$

where $f = f(z) \in \mathbf{R}^m = \mathrm{diag}(f_1(z), f_2(z), \ldots, f_m(z))$ is diagonal nonlinear function and $f_i(z)$ belongs to a sector $[0, \gamma]$.

Assume that only sampled measurements are available for the controller at discrete time instants t_k satisfying

$$0 \leqslant t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$$

with $t_{k+1} - t_k \leqslant h$ and $\lim_{k\to\infty} t_k = \infty$.

In addition, it is assumed that sampled measurement will be quantized before they are transmitted to the controller. In summary, we adopt the following quantized sampled-data feedback controller:

$$\mathcal{C}: u(t) = -Kq(y(t_k) - \hat{y}(t_k)), \quad t \in [t_k, t_{k+1}), \tag{3}$$

where K is the feedback control gain to be determined, and $q(\cdot)$ is a time-invariant and symmetric quantizer.

The purpose of this letter is to design the quantized sampled-data feedback controller (3) with controller gain K such that the synchronization error $e(t) = x(t) - \hat{x}(t)$ converges asymptotically toward zero.

In the following, the quantizer is assumed to be logarithmic and the set of quantized levels is described by Elia and Mitter [18]

$$U = \{\pm u_i : u_i = \rho^i u_0, i = \pm 1, \pm 2, \ldots\} \cup \{\pm u_0\} \cup \{0\},$$

where the parameter $0 < \rho < 1$ is associated with the quantization density.

Then the quantizer q is defined as

$$q(v) = \begin{cases} u_i, & \text{if } v > 0, \text{ and } \frac{u_i}{1+\delta} < v \leqslant \frac{u_i}{1-\delta}, \\ 0, & \text{if } v = 0, \\ -q(-v), & \text{if } v < 0, \end{cases} \tag{4}$$