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Exponential stability analysis of impulsive stochastic functional differential systems with delayed impulses

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ABSTRACT

This paper is concerned with the exponential stability analysis of impulsive stochastic functional differential systems with delayed impulses. Although the stability of impulsive stochastic functional differential systems have received considerable attention. However, relatively few works are concerned with the stability of systems with delayed impulses and our aim here is mainly to close the gap. Based on the Lyapunov functions and Razumikhin techniques, some exponential stability criteria are derived, which show that the system will stable if the impulses' frequency and amplitude are suitably related to the increase or decrease of the continuous flows. The obtained results improve and complement ones from some recent works. Three examples are discussed to illustrate the effectiveness and the advantages of the results obtained.

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1. Introduction

In recent years, the issues of stability and impulsive stabilization of impulsive functional differential systems (IFDSs) which include delay systems have attracted increasing interest in both theoretical research and practical applications (see e.g. [1–9]). In particular, special attention has been focused on the exponential stability of IFDSs and some stability theory have been established (see e.g. [10–15]).

On the other hand, stochastic perturbations is unavoidable in real systems [16]. Stochastic modeling has come to play an important role in many branches of science and engineering. In recent years, an area of particular interest has been stability analysis and impulsive stabilization of impulsive stochastic differential systems (ISDSs) and impulsive stochastic functional differential systems (ISFDSs) (see e.g. [17–31]).

However, to the best of our knowledge, in the previous works on stability of ISFDSs, the authors always suppose that the state variables on the impulses are only related to the present state variables, i.e. $\Delta x(t_k) = I_k(x(t_k^-))$. But in most cases, it is more applicable that the state variables on the impulse are also related to the former state variables, i.e. $\Delta x(t_k) = I_k(x_{t_k^-})$. For example, it is more realistic in practice if the impulsive control depends on a past state due to time lag between the time when the observation of the state is made and the time when the feedback control reaches the system. In fact, there have been several attempts in the literature to study the stability and control problems of a particular class of delayed impulsive systems [32–37]. For example, Lian et al. [32] investigated the optimal control problem of linear continuous-time systems possessing delayed discrete-time controllers in networked control systems. For nonlinear impulsive systems, Khadra et al.

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[33] studied the impulsive synchronization problem coupled by linear delayed impulses. In addition, in [34–36], the authors investigate the asymptotic stability and exponential stability of general IFDSs with delayed impulse (IFDSs-DI). But there are rare results about stability and impulsive stabilization of ISFDSs with delayed impulse (ISFDSs-DI).

The method of Lyapunov functions and Razumikhin techniques are developed to investigate the stability of delay differential systems [38–40], and they have been widely applied to various kinds of systems such as functional differential systems [41], impulsive functional differential systems [8–13], stochastic functional differential systems [16,42–44], infinite dimensional stochastic functional differential systems [45,46], etc.

In this paper, we will extend the Lyapunov–Razumikhin techniques to general ISFDSs-DI. Since an impulsive dynamical system can be viewed as a hybrid one comprised of three components: a continuous-time differential equation, which governs the motion of the dynamical systems between impulsive times; a difference equation, which governs the way in which the system states are instantaneously changed when a resetting event occurs and the connection between these two kind of equations (refer to [47]). So we will divide the systems into two classes: the systems with stable continuous stochastic dynamics and unstable discrete dynamics, the systems with unstable continuous stochastic dynamics and stable discrete dynamics. The first class of systems corresponds to the case when the continuous stochastic dynamics are subjected to impulsive perturbations, while the second class of systems corresponds to the case when impulses are employed to stabilize the unstable continuous stochastic dynamics. We will establish some Razumikhin-type criteria on global exponential stability for each class of systems.

The rest of this paper is organized as follows. Section 2 describes the model of general ISFDSs-DI and introduces the relevant notations. The problem of pth moment exponential stability and almost exponential stability are addressed in Section 3. Section 4 provides three illustrative examples to demonstrate the applications of the obtained results.

2. Preliminaries

Throughout this paper, unless otherwise specified, we let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions, i.e. it is right continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets. Let $w(t) = (w_1(t), \ldots, w_m(t))^T$ be a d-dimensional Brownian motion defined on the probability space. Let $\mathbb{N}$ denotes the set of positive integers, $\mathbb{R}^n$ the n-dimensional real Euclidean space, and $\mathbb{R}^{n\times m}$ the space of $n \times m$ real matrices. I stands for the identity matrix of appropriate dimensions. For $x \in \mathbb{R}^n$, $|x|$ denotes the Euclidean norm. For $A \in \mathbb{R}^{n\times m}$, $\|A\|$ denotes spectral norm of the matrix A. Denote by $\lambda_{min}(\cdot)$ the minimum eigenvalue of a matrix. If A is a vector or matrix, its transpose is denoted by A^T.

Let $\tau > 0$ and $PC([-\tau,0];\mathbb{R}^n)= \{\varphi : [-\tau,0]\to \mathbb{R}^n |\ \varphi(t^+) = \varphi(t)$ for all $t \in [-\tau,0), \varphi(t^-)$ exists and $\varphi(t^-) = \varphi(t)$ for all but at most a finite number of points $t \in (-\tau,0]\}$ be with the norm $\|\varphi\|_\tau = \sup_{-\tau\leqslant\theta\leqslant 0}|\varphi(\theta)|$, where $\varphi(t^+)$ and $\varphi(t^-)$ denote the right-hand and left-hand limits of function $\varphi(t)$ at t respectively. Denote $PC([t_0-\tau,\infty);\mathbb{R}_+) = \{\varphi|\varphi|_{[t_0-\tau,b]} \in PC([t_0-\tau,b];\mathbb{R}_+)$ for all $b > t_0 - \tau\}$

For $p > 0$ and $t \geqslant 0$, let $PC^p_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n)$ denote the family of all $\mathcal{F}_t$-measurable $PC([-\tau,0];\mathbb{R}^n)$-valued random variables φ such that $\sup_{-\tau\leqslant\theta\leqslant 0}\mathbb{E}|\varphi(\theta)|^p < \infty$, where $\mathbb{E}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathbb{P}$. And $PC^b_{\mathcal{F}_{t_0}}([-\tau,0];\mathbb{R}^n)$ denote the family of all $\mathcal{F}_{t_0}$ measurable bounded $PC([-\tau,0];\mathbb{R}^n)$-valued functions.

Consider the following ISFDSs-DI

$$\begin{cases} dx(t) = f(x_t,t)dt + g(x_t,t)dw(t), & t \neq t_k,\ t \geqslant t_0, \\ \Delta x(t_k) = I_k(x_{t_k^-}, t_k), & k \in \mathbb{N}, \end{cases} \tag{2.1}$$

with initial valve $x_{t_0} = \xi = \{\xi(\theta) : -\tau \leqslant \theta \leqslant 0\} \in PC^b_{\mathcal{F}_{t_0}}([-\tau,0];\mathbb{R}^n)$, where $x(t) = ((x_1(t),\cdots,x_n(t))^T$, $x_t = x(t+\theta) \in PC^p_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n)$. Both $f : PC^p_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n) \times \mathbb{R}_+ \to \mathbb{R}^n$ and $g : PC^p_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n) \times \mathbb{R}_+ \to \mathbb{R}^{n\times m}$ are Borel measurable. $I_k(x_{t_k^-}, t_k) : PC^p_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n) \times \mathbb{R}_+ \to \mathbb{R}^n$ represents the impulsive perturbation of x at time t_k. The fixed moments of impulse times t_k satisfy $0 \leqslant t_0 < t_1 < \cdots < t_k < \cdots, t_k \to \infty$ (as $k \to \infty$), $\Delta x(t_k) = x(t_k) - x(t_k^-)$.

As a standing hypothesis, we assume that $f(\varphi,t)$ and $g(\varphi,t)$ are continuous for almost all $t \in [t_0,\infty)$ and are locally Lipschitz in $\varphi \in PC^p_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n)$; $|I_k(\varphi,t)| < \infty$ hold almost surely for all $\varphi \in PC^p_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n)$. According to Theorem 5 in [48], for any $\xi \in PC^b_{\mathcal{F}_{t_0}}([-\tau,0];\mathbb{R}^n)$, there exists a unique stochastic process satisfying system (2.1) denoted by $x(t;t_0,\xi)$, which is continuous on the right-hand and limitable on the left-hand. Moreover, we assume that $f(0,t) \equiv 0, g(0,t) \equiv 0$, and $I_k(0,t) \equiv 0$ for all $t \geqslant t_0, k \in \mathbb{N}$, then system (2.1) admits a trivial solution $x(t) \equiv 0$.

Remark 2.1. The system (2.1) is more general than those investigated in [26–30].

In this paper, we intend to establish sufficient conditions for exponential stability of ISFDSs-DI, which show that the system will stable if the impulses' frequency and amplitude are suitably related to the increase or decrease of the continuous flows.

At the end o f this section, Let us introduce the following definitions.

Definition 2.1. The trivial solution of system (2.1) is said to be

(i) pth ($p > 0$) moment exponentially stable if there is a pair of positive constants λ, C such that

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