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Functional differential inclusions and dynamic behaviors for memristor-based BAM neural networks with time-varying delays ☆

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ABSTRACT

In this paper, we formulate and investigate a class of memristor-based BAM neural networks with time-varying delays. Under the framework of Filippov solutions, the viability and dissipativity of solutions for functional differential inclusions and memristive BAM neural networks can be guaranteed by the matrix measure approach and generalized Halanay inequalities. Then, a new method involving the application of set-valued version of Krasnoselskii' fixed point theorem in a cone is successfully employed to derive the existence of the positive periodic solution. The dynamic analysis in this paper utilizes the theory of set-valued maps and functional differential equations with discontinuous right-hand sides of Filippov type. The obtained results extend and improve some previous works on conventional BAM neural networks. Finally, numerical examples are given to demonstrate the theoretical results via computer simulations.

1. Introduction

As is well known, the discontinuous or non-Lipschitz dynamical system has become an old research topic for several decades. This class of discontinuous dynamical systems is usually caused by natural phenomenon or control actions of many interesting engineering tasks, and can be described by the first order ordinary differential equations (ODE) or functional differential equations (FDE) with discontinuous right-hand sides. Important examples include static friction, impacting machines, contacting collision, switching in electronic circuits, variable structure control and many others (see [1–5]). As a special kind of dynamical system, discontinuous dynamical neuron systems have been extensively and intensively studied since their wide applications during the past few decades, such as associative memory, pattern recognition, image and signal processing, automatic control, quadratic optimization, parallel computing, and so on (see, for example, [6–12]). It is worth noting that neural networks can be implemented by very large-scale integration (VLSI) circuits. However, when the switching occurs in many practical implementing and applications of neural networks, the dynamics will be defined by discontinuous vector field. Take the classic bidirectional associative memory (BAM) neural networks as an example, this class of neural network model can be implemented in a circuit where the self-feedback connection weights and the connection weights are implemented by resistors. Suppose that we replace the resistors with memristors, then we can build a new BAM neural network model (memristor-based BAM neural network) where the parameters change according to its state, that is, it is a state-dependent switching BAM neural network dynamical systems. In this case, this memristor-based BAM

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neural network system can be described by ODE or FDE possessing discontinuous right-hand side. Moreover, there will emerge some specially interesting and important dynamical traits that are not captured by the continuous system, e.g., the presence of sliding modes along discontinuity surfaces and the phenomenon of convergence in finite time toward the equilibrium point or periodic solution (limit cycle).

Actually, the research on memristor-based neural networks began in the early 1970s. Professor Chua [13] is the first one to introduce and name memristor (an abbreviation for memory resistor) in 1971. However, such a memristor only is an ideal circuit element based purely on symmetry arguments. Fortunately, a more practical memristor device had been proposed by scientists although no much attention was paid to Chua's theory until nearly 40 years later. On 1 May 2008, members of Hewlett–Packard (HP) Lab announced that they had built a prototype of the memristor based on nanotechnology in Nature [14,15]. This new circuit element is a two-terminal element with variable resistance called memristance which shares the same unit of measurement (ohm) and possesses many properties of resistors (e.g., memory characteristic and nanometer dimensions). It should be pointed out that the memristor exhibits the feature of pinched hysteresis just as the neurons in the human brain have [16–18]. Owing to this important feature, we can apply this device to design some new models of neural networks for emulating the human brain, and enables us to further study the dynamical behaviors for understanding the function of human brain. Up to now, memristor has received widespread concern from many scientists because of its potential applications in next generation computer and powerful brain-like "neural" computer (see, for example, [16–25]). Nevertheless, existing memristor-based neural networks designed by many researchers have been found to be computationally restrictive. Furthermore, there still lacks new and efficacious methods for the qualitative analysis and stability analysis of memristor-based neural network dynamical systems. We can see that many excellent results for various memristor-based neural networks have been obtained by using the new framework of differential inclusion in recent years (see, for example [26–30] and the references cited therein). To the best of our knowledge, there is not much research concerning memristor-based bidirectional associative memory neural networks and usually neglects the effect of periodicity in the dynamic analysis of many memristive neuron systems. This directly motivates us to explore some complex dynamical properties of memristor-based dynamical neuron systems such as memristive and time-varying delayed BAM neural networks with periodic or almost periodic inputs.

It is worth mentioning that a large number of basic questions arise when discontinuities occur in a dynamical system defined by some differential equation. The most fundamental and natural issue is about the solution of discontinuous dynamical systems. Obviously, the existence of a continuously differentiable solution is not guaranteed for such discontinuous systems. That is to say, the classical solutions might not exist if the vector field is discontinuous (see [2]). However, what is new analysis and synthesis framework for the solutions of discontinuous systems? And how to ensure the global existence, uniqueness and convergence of such solutions? It should be noted that in many cases, we have dealt with the switching or discontinuous systems without caring these questions. Fortunately, the theory of differential inclusion with the new framework of Filippov solution has been established and has become a standard and effective tool to deal with the problems of dynamical behaviors for discontinuous systems (or differential equations with discontinuous right-hand sides). In fact, by constructing the Filippov set-valued map, the solution of differential equation could be transformed into a solution of differential inclusion, which is also called as the Filippov regularization. Such a concept in the sense of Filippov has been utilized as a feasible approach in the field of mathematics and control especially for discontinuous dynamical neuron systems, and has been accepted universally as a good one to investigate the switching or discontinuous dynamical systems whose right-hand sides only required to be Lebesgue measurable in the state and time variables. As shown by Lu and Chen [10], we can also model a system that is a near discontinuous system and expect that the Filippov trajectories of the discontinuous system will be close to the trajectories of actual system since a Filippov solution is a limit of solutions of ordinary differential equation with a continuous right-hand side. Generally speaking, any mathematical model of the dynamic system containing uncertainties can be represented as a differential inclusion equation because uncertainties may lead to a suddenly change of a trajectory in such a dynamic system. Indeed, differential inclusion system is considered as a generalization of the system described by differential equation. Moreover, in the process of simplification of many practical problems, differential inclusion theory can relax some restrictions to the utmost extent, and does not affect the essence of the problems. On the other hand, time delays in neuron signals are often inevitable due to internal or external uncertainties in many practical application of neural networks. As pointed out by [31–33], in electronic implementation of artificial neural networks, the time delays between neurons are usually time variant, and sometimes vary dramatically with time because of the finite switch speed of amplifiers and faults in the electrical circuits. From the theoretical view, when the time delays are introduced into the discontinuous dynamical neuron systems, the theory of functional differential inclusions (i.e., differential inclusions with memory) is used as a useful tool to analyze the solutions and further to explore the dynamical behaviors for such discontinuous neuron systems modeled by functional differential equations with discontinuous right-hand sides. As discussed by Aubin and Cellina [34], functional differential inclusions express that the velocity depends not only on the state of the system at every instant, but depends upon the history of the trajectory until this instant. Therefore, it is of practical importance and interesting to investigate the discontinuous or switching dynamical neuron systems via differential inclusions or functional differential inclusions with the Filippov-framework.

The structure of this paper is outlined as follows. In Section 2, the model formulation and some preliminaries including some necessary definitions and lemmas are presented. In Section 3, the global existence of Filippov solutions for functional differential inclusions with given initial conditions is considered and some sufficient conditions are given to guarantee the global dissipativity by using matrix measure approach. In Section 4, we discuss the existence of positive periodic solutions