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Specific quantum mechanical solution of difference equation of hyperbolic type

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ABSTRACT

Difficulties connected to solving difference equations of hyperbolic type were analyzed in this work and discussed in detail. The results are compared to those of the standard wave equation and certain similarities were established. The method of solving the equation is generalized by means of kernel expanded into separable polynomials. The analysis was inspired by some new ideas concerning quantization of time. Two examples are given: excitons and phonons in thin crystalline films. The advanced methodology of Green's function method and the application of this new methodology resulted in a set of interesting conclusions concerning thin film properties. The significance of the obtained spatial dependence of exciton concentration was discussed and it was concluded, on the basis of the found spatial dependence of exciton concentration, that such boundary conditions of a thin molecular film which lead to high exciton concentrations can be determined. It was also concluded that thin films possess high superconductive properties, that physical characteristics of thin films are spatially dependent and that the spatial dependence can be the basis for widening the field of application of nanostructures.

1. Background

Standard hyperbolic equation describing vibration processes in linear structures contains second derivatives with respect to time and with respect to spatial coordinate. In practical problems it was used mostly in analyses of musical instruments. In recent time the interest for this equation grows since the Schrödinger equation for particles and quasiparticles with zero rest mass (photons, phonons, neutrinos, etc.) translates from parabolic type to hyperbolic one.

The difference equivalence of hyperbolic differential equation can find very wide field of application.

Considering the vast empty space between atoms of solid state crystal lattice, application of continual spatial coordinates represents just a mathematical idealization. Especially when nanostructures are considered. The problem of time variable remains open. In science two approaches exist. On one hand, according to Aristotle and Newton, time is continual and untouchable (the flow of time cannot be influenced). On the other hand, according to Tit Lucretius Car and Albert Einstein, time does not exist without events [1,2]. Both of these comprehensions are possibly correct, since the concept of time is widely used in life and science. We shall accept the Car-Einstein concept and consider time as a reciprocal frequency value. Such determination of time makes it a dynamical variable (it can be influenced by force). The time defined in this way will be hereinafter called eigentime or self-time.

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In the first section it will be demonstrated that eigenvalues of eigentime can be elements of numerable sets. This proof legalizes formulation of the difference hyperbolic equation in the second section. The method of solving this equation is generalized in the third section by means of kernel expanded into separable polynomials.

2. Eigentime as quantum mechanical operator

In the introductory part, we defined eigentime as a reciprocal value of frequency of the system. In accordance with this definition, eigentime is a dynamical variable (i.e. it can be influenced by force). The operator of eigentime will be determined from Schrödinger equation which is the dynamical law of quantum mechanics [2–4]. From Schrödinger equation for free particle follows an equivalency:

$$i\hbar\frac{\partial}{\partial t} \Longleftrightarrow -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} \tag{2.1}$$

So, the operator of eigentime will be determined as

$$\hat{\tau} = h\left(i\hbar\frac{\partial}{\partial t}\right)^{-1} = \frac{2\pi}{i}\int dt. \tag{2.2}$$

It should be noted that integration constant appearing in integration must be equal to zero since inverse operators are connected by the relation: $\frac{d}{dt}\int dt = \int dt\frac{d}{dt}$.

The material waves described by Schrödinger equation must have their time period T. It is the reason to introduce an operator of time period $\hat{T}$. Using equivalency (2.1) we shall define the operator as

$$\hat{T} = h\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)^{-1} = \frac{8m\pi^2}{h}\int dx\int dx. \tag{2.3}$$

Eigenvalues of operator $\hat{T}$ will be denoted by T and will be the time period of material waves.

Now, we solve eigenproblem of eigentime, denoting eigenvalues of eigentime by Θ:

$$\frac{2\pi}{i}\int dt\psi(t) = \Theta\psi(t). \tag{2.4}$$

After differentiation with respect to time, we obtain

$$\frac{d\psi}{dt} = -\frac{2\pi i}{\Theta}\psi, \tag{2.5}$$

wherefrom it follows:

$$\psi(t) = Ce^{-\frac{2\pi i}{\Theta}t}. \tag{2.6}$$

The solution (2.6) must be unique. It means that the condition $\psi(t) = \psi(t+T)$ must be satisfied, which is fulfilled for eigenvalues:

$$\Theta_n = \frac{T}{n} = \frac{2mh}{p^2}\frac{1}{n}; \quad n = 1,2,3,\ldots \tag{2.7}$$

It is seen that eigenvalues Θ_n of the operator $\hat{\tau}$ are elements of numerable sets, i.e. eigentime can be quantized. The demonstration shows that time, taken as reciprocal frequency, can be treated as a dynamical variable with discrete spectrum. In the formula (2.7) value $n = 0$ is formally allowed. Since n characterizes frequency, the value corresponds to the state when nothing happens, i.e. vibrations are absent. For this value of n, time Θ_n becomes infinite, which is compatible with Einstein's idea: time does not exist without events. We will not analyze properties of time out of scope of the mentioned definition.

3. Hyperbolic difference equation

The hyperbolic differential equation of mathematical physics

$$\frac{\partial^2\Psi}{\partial t^2} - c^2\frac{\partial^2\Psi}{\partial x^2} = 0, \tag{3.1}$$

is often used and investigated in detail [5,6], where the propagation velocity of the waves is denoted by c. However, the corresponding difference equation

$$\frac{\Delta^2\Psi_{m,n}}{\Delta m^2} - c^2\frac{\Delta^2\Psi_{m,n}}{\Delta n^2} = 0, \tag{3.2}$$