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Multi-objective optimal design of feedback controls for dynamical systems with hybrid simple cell mapping algorithm

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ABSTRACT

This paper presents a study of multi-objective optimal design of full state feedback controls. The goal of the design is to minimize several conflicting performance objective functions at the same time. The simple cell mapping method with a hybrid algorithm is used to find the multi-objective optimal design solutions. The multi-objective optimal design comes in a set of gains representing various compromises of the control system. Examples of regulation and tracking controls are presented to validate the control design.

1. Introduction

Full state feedback control is an important part of the modern control theory. Because of its vast applications in industries, there have been many studies to develop design or tuning techniques of the control. The well-known linear quadratic regulator (LQR) is the most popular optimal controller design in the modern control theory [1]. Since feedback controls are often designed to meet multiple and possibly conflicting performance goals, comprehensive studies are usually carried out to tune control gains in order to achieve the best overall performance [2,3]. This paper presents an efficient numerical method for designing multi-objective optimal feedback controls.

In the last three decades, there have been a large number of publications on multi-objective optimal design of feedback controls. Different from the traditional single objective optimization problems (SOPs), the multi-objective optimization problems (MOPs) no longer have unique solutions consisting of a single point in the design space, but rather a set, called *Pareto set*. The corresponding objective function values are called *Pareto front*.

Multi-objective optimal control design can be carried out in time domain or frequency domain. Time domain approach uses the time domain specifications of the closed-loop response as the objective functions such as overshoot, peak time, settling time and tracking error [4]. On the other hand, frequency domain design uses phase and gain margins as the objectives, and can consider robust issues such as model uncertainty, load disturbance and measurement noise. Multi-objective optimization with robustness often involves the optimization among several norms. Vroemen and Jager reviewed the

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multi-objective design of robust controls for linear systems [5]. They examined different combinations of $\mathcal{H}_2, \mathcal{H}_\infty$ and L_2 norms to formulate the robust control synthesis problems. A more recent overview by Gambier and Badreddin summarized most available methods for multi-objective optimal control design in both time and frequency domain [6]. They stated that despite the significant development of multi-objective optimization in control engineering, on-line design methods with multi-objective optimization are still at the beginning phase.

Even though there have been many studies of multi-objective optimization control design for linear systems, only a handful references are available for nonlinear systems, and are scattered in different disciplines. Since the concept of frequency domain in nonlinear system is not as well studied as in linear systems, the control design for nonlinear systems was usually done in time domain. A nonlinear fuzzy controller based on Pareto rule-base design is carried out by examining the temporal response in [7]. A variable complexity modelling technique with multi-objective optimization design was studied by Silva et al. to tune the multivariable PI control of a nonlinear thermodynamic model in gas turbine. A more theoretical research of multi-objective nonlinear control is presented in reference [8] where the multi-objective optimization algorithm is combined with the classical variational method.

Many algorithms for obtaining the Pareto set and Pareto front of MOPs have been developed. There are biologically inspired optimization algorithms such as Genetic Algorithm [9], Ant Colony Optimization [10], Immune Algorithm [11] and Particle Swarm Optimization [12]. All these methods have been successfully applied to feedback control design including PID controls to meet multiple objectives. Fliege and Saviter have developed several gradient-based algorithms by converting MOP to SOP for point-wise evolution and step length determination of the steepest descend search for MOP solutions [13]. Peter [14] expands the concept of gradient by introducing novel geometric transformations and combines it with the Genetic Algorithm for MOPs. A gradient-free approach is introduced by Zhong et al. [15] to address MOPs with undifferentiable objective functions. In the work of Custodio et al. [16], methods for pattern searching are adopted to direct gradient-free search.

Another approach to find the Pareto set is to use the set oriented methods with subdivision techniques [17–19]. The advantage of the set oriented methods is that they generate an approximation of the global Pareto set in one single run of the algorithm. The cell mapping method in this study is the predecessor of the set oriented methods, and was proposed by Hsu [20] for global analysis of nonlinear dynamical systems. Two cell mapping methods have been extensively studied, namely, the simple cell mapping (SCM) and the generalized cell mapping (GCM) to study the global dynamics of nonlinear systems [21,20]. The cell mapping methods have been applied to optimal control problems of deterministic and stochastic dynamic systems [22–24]. Other interesting applications of the cell mapping methods include optimal space craft momentum unloading [25], single and multiple manipulators of robots [26], optimum trajectory planning in robotic systems by [27], tracking control of the read-write head of computer hard disks [28], and airfoil flutter analysis [29]. Sun and his group studied the fixed final state optimal control problems with the simple cell mapping method [30,31], and applied the cell mapping methods to the optimal control of deterministic systems described by Bellman's principle of optimality [32]. Recently, we have found that the SCM method can discover the global Pareto fronts with fine structures in a quite effective manner for low and moderate dimensional problems [33,34].

This paper studies multi-objective optimal full state feedback controls. The hybrid algorithm taking advantages of both gradient-based and gradient-free searching strategies [34] is presented to investigate high dimensional MOP design of the full state feedback control. A regulation and a tracking problem are studied to demonstrate the effectiveness of the control design.

The rest of this article is outlined as follows. Section 2 defines the multi-objective optimization problem (MOP). Section 3 presents the gradient-based and gradient-free algorithm for MOP solution with the SCM method. Section 4 applies the proposed method to design full state feedback controls for regulation and tracking of two degree of freedom systems. We close the paper in Section 5 with concluding remarks.

2. Multi-objective Optimization

A multi-objective optimization problem (MOP) can be stated as follows:

$$\min_{\mathbf{k}\in Q}\{\mathbf{F}(\mathbf{k})\}, \tag{1}$$

where

$$\mathbf{F}(\mathbf{k}) = [f_1(\mathbf{k}),\ldots,f_k(\mathbf{k})], \quad f_i : Q \to \mathbf{R}^1,\ \mathbf{F} : Q \to \mathbf{R}^k. \tag{2}$$

f_i are objective functions, $\mathbf{k} \in Q$ is a q-dimensional vector of design parameters. The domain $Q \subset \mathbf{R}^q$ can in general be expressed in terms of inequality and equality constraints,

$$Q = \{\mathbf{k} \in \mathbf{R}^q \mid g_i(\mathbf{k}) \le 0,\ i = 1,\ldots,l,\ \text{and}\ h_j(\mathbf{k}) = 0,\ j = 1,\ldots,m\}. \tag{3}$$

Next, we define optimal solutions of the MOP by using the concept of *dominance* [35].