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Stochastic stability analysis for joint process driven and networked hybrid systems ☆

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ABSTRACT

The stochastic stability and impulsive noise disturbance attenuation in a class of joint process driven and networked hybrid systems with coupling delays (JPDNHSwD) has been investigated. In particular, there are two separable processes monitoring the networked hybrid systems. One drives inherent network structures and properties, the other induces random variations in the control law. Continuous dynamics and control laws in networked subsystems and couplings among subsystems change as events occur stochastically in a spatio-temporal fashion. When an event occurs, the continuous state variables may jump from one value to another. Using the stochastic Lyapunov functional approach, sufficient conditions on the existence of a remote time-delay feedback controller which ensures stochastic stability for this class of JPDNHSwD are obtained. The derived conditions are expressed in terms of solutions of LMIs. An illustrative example of a dynamical network driven by two Markovian processes is used to demonstrate the satisfactory control performance.

1. Introduction

Hybrid systems have been of interest in the literature since they have many important applications in, for example, robotics, communication systems, chemical processes, and so on. Such systems consist of both continuous variables and discrete events. Models of hybrid systems have been proposed in various formalism [15], including hybrid automata [1] and hybrid machines [11–14]. Progresses in modeling and control of hybrid systems can be found in books and literatures [5,7,17,29].

In recent years, most hybrid systems were analyzed as standalone devices, but many practical systems exhibit their dynamical processes as a network of interacting components. Complexity of the resultant network is derived from topological structure, network evolution, connection and node diversity, and/or dynamical evolution [2]. Moreover, the variations of a networked hybrid system are often monitored by one process whose behavior is conditioned by the other process in order to change the control accordingly. Thus, we call such a hybrid system *joint process driven and networked hybrid system* (JPDNHS). For instance, a fault tolerant control system can be modeled as failures with markovian transition characteristics;

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and an additional random process is a failure detection and identification (FDI) process, which changes the system description through the control reconfiguration strategy [23].

In this paper, the networked hybrid system is modeled by a hybrid machine, because hybrid machines with their explicit inputs and outputs are particularly suitable for control. The hybrid machine consists of a set of vertices (discrete states). At each vertex, the continuous dynamics is described by vertex-dependent state equations, which is composed of a set of subsystems (called nodes) that are coupled together. Both (continuous) dynamics and coupling among subsystems change as events occur. We assume that events are triggered by an environment and/or a control strategy in a spatio-temporal random fashion, which means the random process depends upon both time instant t and the index of node i. That is to say, for different node the event transition matrix is different. When an event occurs, the continuous state variables may jump from one value to another, since the noise on the channel is impulsive in nature due to a noisy sensor or channel transmission errors.

The uncontrolled system and the control law monitored by two separable markov processes has random jumping parameters and delays. We use a time-varying delay state feedback to control the JPDNHSwD. One typical example is controlling intelligent robots through the Internet. To ensure stochastic stability and disturbance attenuation, stochastic Lyapunov functions are used. Sufficient conditions on stochastic stability are all derived in terms of solutions of LMIs. An example of JPDNHSwD with twenty nodes is used to illustrate the results.

The main reason behind using the JPDNHSwD model is to make it more suitable and practical in representing the following aspects. (1) It allows a joint process monitoring system. That is, the system description depends on the state of one process Pro_I while the input applied to the system depends on the control law in response to the other process Pro_{II}. (2) It allows events generated by the environment and/or the control in a spatio-temporal random switching fashion. (3) It allows both continuous and discrete dynamics. In particular, continuous variables can be re-initialized when transitions from one vertex to another occur in the control strategy. Therefore, trajectories of the systems can exhibit jumps. (4) It allows the networked subsystems to be controlled in a decentralized and distributed fashion. In particular, remote feedback control with impulsive noise disturbance is considered. Thus, transmitting delays exist in the controller signal and impulsive noise. (5) It allows both event transitions and dynamical transitions triggered by continuous dynamics. (6) It allows time-varying delays between subsystems. Some individual aspects listed above have been studied in the literature. (3) is studied in [7,18,25,27,32] using an impulsive framework. (4) is studied in [21,30] using Internet-based control systems. (5) is studied in [4,9,16,22,24] in the framework of stochastic hybrid systems. (6) is studied in [6,28,26] using Markovian jump systems. However, to the best of our knowledge, no paper has addressed all six aspects listed above. The inadequacy of the existing results in analyzing the stability of JPDNHSwD is the primary motivation of this paper. In particular, no paper has addressed (1) and (2). We will examine the stochastic stability of joint process driven and networked hybrid systems in which events are generated in a spatio-temporal random fashion.

Although we consider only two separable processes to drive the system dynamics and control law, our approach is general and paves the way for future extension of this method to more complex and practical problems. Our approach also extends Markovian jump systems and connects the work on Markovian jump systems to work on hybrid systems and hence provides a fresh new look at the problem. In our previous paper [31], we study H_∞ control for stochastic stability and disturbance attenuation in a class of networked hybrid systems. In that paper, we consider only event transitions that are generated by the environment in a Markovian fashion. The trajectories of the systems can exhibit jumps when transitions from one vertex to another occur in the system dynamics. We consider only one process driven and networked hybrid systems. All these three aspects are different from what we consider in this paper.

This paper is organized as follows. In Section 2, we first briefly review the hybrid machine model of hybrid systems, and then present joint process driven and networked hybrid systems with delays to be investigated. The stochastic stability and impulsive disturbance attenuation problem to be solved is formulated in Section 3. A numerical example to illustrate the results is presented in Section 4. Finally, some conclusions are drawn in Section 5.

2. Problem formulation

Let $C^{2,1}(R^n \times Q; R^+)$ denote the family of all nonnegative functions $v(x,q)$ on $R^n \times Q$. The notation $A > 0\,(<0)$ is used to denote a positive (negative) definite matrix. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ represent the minimum and maximum eigenvalues of the corresponding matrix, respectively. $\mathbf{E}[\cdot]$ denotes the mathematical expectation. The identity matrix of order n is denoted as I_n (or simply I if no confusion arises). For $x = (x_1, \ldots, x_n)^\top \in R^n$, the norm of x is $\|x\| := (\sum_{i=1}^n x_i^2)^{\frac{1}{2}}$ and $|x| := (|x_1|, \ldots, |x_n|)^\top$. Correspondingly, for $A = (a_{ij})_{n\times n} \in R^{n\times n}$ and $B = (b_{ij})_{n\times n} \in R^{n\times n}$, $\|A\| := \lambda_{\max}^{\frac{1}{2}}\left(A^T A\right)$ and $|A| := (|a_{ij}|)_{n\times n}$. We use the notations $A \geqslant B$ and $(x_1, \ldots, x_n)^\top \geqslant (y_1, \ldots, y_n)^\top$ to imply that $a_{ij} \geqslant b_{ij}$ and $x_i \geqslant y_i, i,j = 1, \ldots, n$ respectively.

2.1. Hybrid machine model

We first review a modeling formalism for a class of hybrid systems which we call hybrid machines [11–13]. An elementary hybrid machine (EHM) is denoted by

$$EHM = (Q, \Sigma, Dy, E, (q_0, x_0)).$$