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Exponential synchronization of discontinuous chaotic systems via delayed impulsive control and its application to secure communication

Xinsong Yang [a,*], Zhichun Yang [a], Xiaobing Nie [b]

[a] Department of Mathematics, Chongqing Normal University, Chongqing 401331, China
[b] Department of Mathematics, Southeast University, Nanjing 210096, China

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ABSTRACT

This paper investigates drive-response synchronization of chaotic systems with discontinuous right-hand side. Firstly, a general model is proposed to describe most of known discontinuous chaotic system with or without time-varying delay. An uniform impulsive controller with multiple unknown time-varying delays is designed such that the response system can be globally exponentially synchronized with the drive system. By utilizing a new lemma on impulsive differential inequality and the Lyapunov functional method, several synchronization criteria are obtained through rigorous mathematical proofs. Results of this paper are universal and can be applied to continuous chaotic systems. Moreover, numerical examples including discontinuous chaotic Chen system, memristor-based Chua's circuit, and neural networks with discontinuous activations are given to verify the effectiveness of the theoretical results. Application of the obtained results to secure communication is also demonstrated in this paper.

1. Introduction

Dynamical behaviors of systems which are described by differential equations with discontinuous state on the right-hand sides have attracted increasing attention of researchers in the past decades since they have many important applications involving impacting machines, dry friction, systems oscillating under the effect of an earthquake, switching in electronic circuits and so on [1–3]. However, due to the discontinuity of the state on the right-hand sides of differential equations, solutions in the classical sense to this type systems may not exist. In order to study the dynamical behaviors of such type systems, several solution notions such as Caratheodory solutions [4], Krasovskii solutions [5], and Filippov solutions [4] have been proposed. Although different person may have different solution notion, the Filippov solution is widely used in today's academia. By using the Filippov regularization method proposed in [4], a differential equation with discontinuous right-hand side can be transformed into a differential inclusion. Based on the theories of differential inclusion, many results concerning the existence and stability of solutions to differential equations with discontinuous right-hand side have been obtained [6–12], but relatively, results on synchronization of differential equations with discontinuous right-hand sides are few.

Synchronization, which means that the dynamical behaviors of coupled systems achieve the same time-spatial property, can be found in a wide variety of application fields such as secure communication, biological systems, information processing [13–16]. Since the pioneering work of Pecora and Carroll [17], chaos synchronization and control have intrigued increasing interest of researchers in different disciplines. As far as we know, most of known results on chaos synchronization and

* Corresponding author. Tel./fax: +86 23 65362798.
E-mail addresses: xinsongyang@163.com (X. Yang), yangzhch@126.com (Z. Yang), xbnie@seu.edu.cn (X. Nie).

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control are about continuous chaotic systems. In fact, discontinuous dynamical systems can also exhibit chaotic behaviors, for example, discontinuous Sprott circuit [18], discontinuous Chua's circuit [19], memristor-based Chua's circuit [20], discontinuous Chen system [21], and neural networks with discontinuous activation functions [22,23]. Because of the uncertainty of the directional derivatives of Filippov solutions on the discontinuous surface, results on stability of differential inclusions can not be simply extended to synchronization of differential inclusions (see Remark 4).

Recently, some attempts have been devoted to chaotic synchronization and control of discontinuous chaotic systems [20,22–26]. In [20], complete synchronization of memristor-based Chua's circuit was studied by using state feedback control, but the memductance function must be included in the state feedback controller. Authors of [25] proposed a "weak-QUAD" and a "semi-QUAD" conditions for chaotic systems with discontinuous right-hand side, under which several synchronization criteria were obtained for complex networks. But the "weak-QUAD" and "semi-QUAD" conditions on the discontinuity of discontinuous function were weakened, i.e., as time goes to infinity, the discontinuous function approaches to a continuous function. As for neural networks with discontinuous activation functions, quasi-synchronization criteria were obtained in [22,24,23]. Asymptotic synchronization of delayed neural networks with discontinuous activations was investigated in [27] via adaptive control. But the discontinuous activation functions in [27] are assumed to be monotonic. In [26], exponential synchronization of a class of memristor-based recurrent neural networks was studied, but the condition (A3) in [26] is not correct. Moreover, results in the above mentioned papers are only applicable to some specific models. To the best of the authors' knowledge, no result concerning complete synchronization for general discontinuous chaotic systems is published to date, and this is the main motivation of the present contribution. In order to do this, new universal conditions and new controllers need to be designed.

Along with the investigation of chaos synchronization, many effective control methods have been developed, for instance, state feedback control [28,29], adaptive control [27,30], impulsive control [31]. Among the different control techniques, impulsive control method is attractive because it needs small control gain and acts only at discrete times, thus control cost and the amount of transmitted information can be reduced drastically. Although there have been a lot of previous works concerning synchronization of continuous chaotic systems by using impulsive control in the literature (see [14,32–35], and references cited therein), few results, however, are published on synchronization of discontinuous chaotic systems via impulsive control. Note that the impulsive controllers in [14,32–35] were non-delayed. Since time-delays inevitably appear when signals are transmitted from the receiver to the controlled system, it is necessary to consider time-delay in controllers. Although many published papers considered time-delays in state feedback controllers [36–38], few published paper consider time-delay in impulsive controller. Recently, in [31] we gave a lemma on impulsive differential inequality. Based on the given lemma, we successfully synchronized a class of reaction–diffusion neural networks by using impulsive controller with and without time-varying delays. However, the time-delayed impulsive controller in [31] cannot be simply extended to realize synchronization of discontinuous chaotic systems, so it is urgent and important to design new impulsive controller with time-varying delays to synchronize general discontinuous chaotic dynamical systems.

Motivated by the above analysis, this paper aims to investigate global exponential synchronization for a class of discontinuous chaotic dynamical systems by using delayed impulsive control. The considered model includes time-varying delay and covers most of known continuous and discontinuous chaotic systems. By using the sign function, suitable impulsive controller, which is practical thanks to multiple unknown time-varying delays, is designed to achieve the synchronization goal. By using the Gronwall inequality and matrix theory, sufficient conditions ensuring the existence of solution in the sense Filippov concept are firstly derived. Based on the lemma on impulsive differential inequality obtained in [31] and the Lyapunov functional method, two global exponential synchronization criteria are derived through rigorous mathematical proofs. The synchronization conditions derived in this paper can also be applied to continuous chaotic systems. Moreover, three different discontinuous chaotic systems are employed to verify the effectiveness of the theoretical results. Application of the obtained results to secure communication is also demonstrated in this paper. Theoretical proofs and numerical simulations demonstrate that the results of this paper are new and some existing ones are extended and improved.

The rest of this paper is organized as follows. In Section 2, model of general discontinuous systems with delay is described. Some necessary assumptions, definitions and lemmas are also given in this section. Exponential synchronization of the considered model under delayed impulsive control are studied in Section 3. In Section 4, numerical simulations are given to show the effectiveness of our results. In Section 5, conclusions are given.

Notations: In what follows if not explicitly stated, matrices are assumed to have compatible dimensions. $\mathbb{N}_+$ is the set of positive integers, I_m denotes the identity matrix of m-dimension. $\mathbb{R}$ is the space of real number. The Euclidean norm in $\mathbb{R}^m$ is denoted as $\|\cdot\|$, accordingly, for vector $x \in \mathbb{R}^m$, $\|x\| = \sqrt{x^T x}$, where T denotes transposition. $x = 0$ represents each component of x is zero. $A = (a_{ij})_{m\times m}$ denotes a matrix of m-dimension, $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$.

2. Preliminaries

Generally, a discontinuous chaotic system with time-varying delay is described as follows:

$$\dot{x}(t) = Cx(t) + Af(x(t)) + Bh(x(t-\tau(t))), \tag{1}$$