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Trajectory tracking control of variable length pendulum by partial energy shaping

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ABSTRACT

This paper concerns a trajectory tracking control problem for a pendulum with variable length, which is an underactuated mechanical system of two degrees-of-freedom with a single input of adjusting the length of the pendulum. We aim to study whether it is possible to design a time-invariant control law to pump appropriate energy into the variable length pendulum for achieving a desired swing motion (trajectory) with given desired energy and length of the pendulum. First, we show that it is difficult to avoid singular points in the controller designed by using the conventional energy-based control approach in which the total mechanical energy of the pendulum is controlled. Second, we present a tracking controller free of singular points by using only the kinetic energy of rotation and the potential energy of the pendulum and not using the kinetic energy of the motion along the rod. Third, we analyze globally the motion of the pendulum and clarify the stability issue of two closed-loop equilibrium points; and we also provide some conditions on control parameters for achieving the tracking objective. Finally, we show numerical simulation results to validate the presented theoretical results.

1. Introduction

Studying pendulum(s) is one of the classical problems of mechanics [1,2] and pumping a swing is a classical problem that has been widely studied and various pumping strategies have been proposed [3,4]. The child's swing is usually modeled as a variable length pendulum (VLP), and control is accomplished by changing the length such as standing, seating, and squatting to generate or stabilize the oscillation of the pendulum, see, e.g. [3,4]. Regarding a kip motion of an expert gymnast on the high bar, Nakawaki et al. [5] showed that a VLP model is sufficient for modeling the dynamics of the center of mass of the gymnast without considering the complex parameters of the 3-link model of the gymnast.

On the other hand, a VLP is also an example of underactuated mechanical systems with two degrees-of-freedom (the angular displacement and the length of the pendulum) with a single input to adjust the length of the pendulum. The VLP is uncontrollable at vertical position since the angular displacement of the pendulum can not be controlled by adjusting its length at vertical line. Recent years many researchers have made considerable efforts in studying underactuated mechanical systems, see, e.g. [6–13]. For an underactuated mechanical system described by the Euler–Lagrange equations, [14,15] addressed the control problem of aiming to stabilizing its total mechanical energy to a desired value and stabilizing its actuated variable(s).

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Pumping a swing is a classical problem that has been widely studied and various pumping strategies have been proposed [3,4]. Many time-varying laws of changing the length of the pendulum are proposed, and the stability and instability of periodic solutions of the resulting time-varying systems are analyzed by using the nonlinear oscillation theory. For example, for a pendulum with periodically varying length, [1] derived asymptotic expressions for boundaries of instability domain near resonance frequencies by the average method; [16] investigated periodic boundary value problems that arise in analyzing the stability of the lower position of equilibrium of a VLP. In [17], the authors studied the swing-up and the stabilization at the upright position of the free second link in a two-link underactuated manipulator with actuated first link and free second link; indeed, the actuated first link is controlled by high-frequency excitation and the bifurcations produced in the free second link are used to realize the control objective.

Different from these results, inspired by recent progress in the energy-based control approach for controlling underactuated mechanical systems in [14,15], this paper aims to study whether it is possible to design a time-invariant control law to pump appropriate energy into the VLP for achieving a desired swing motion (trajectory) with given desired energy and length of the pendulum.

First, we show that it is difficult to avoid singular points in the controller designed by using the conventional energy-based control approach proposed in [14,15], in which the total mechanical energy of the system is controlled. Second, we present a controller free of singular points by proposing a new Lyapunov function which shapes a part of the total mechanical energy including the kinetic energy of rotation and the potential energy of the VLP; that is, the kinetic energy of the motion along the rod is not utilized. Third, we present a global motion analysis of the VLP and provide some conditions on control parameters for achieving the trajectory tracking objective and investigate the stability of two closed-loop equilibrium points.

Specifically, we show that the downward equilibrium point, where the VLP is at the downward position, is not hyperbolic. Indeed, its corresponding Jacobian matrix has two eigenvalues on the imaginary axis with the rest of the eigenvalues having negative real parts. Therefore, linearization fails to determine the stability of such an equilibrium point [18]. This fact is different from the stability analysis of the Acrobot in [19] for which the Jacobian matrix valued at the downward equilibrium point is shown to have at least one eigenvalue with positive real part. Note that the Acrobot is a two-link planar robot with a single actuator at the second (elbow) joint of the robot. In this paper, we show that the downward equilibrium point of the VLP is unstable in the sense of Lyapunov by using some features of the newly presented Lyapunov function.

This paper is organized as follows: Section 2 presents some preliminary knowledges about the VLP and describes the tracking control problem to be solved. Section 3 addresses the designs of the tracking controller for the VLP. Section 4 analyzes the global motion of the VLP under the presented controller and reveals instability of its downward equilibrium point. Section 5 discusses the effect of damping on the motion of the VLP. Section 6 shows simulation results for a VLP for validating the presented theoretical results. Section 7 makes some concluding remarks.

2. Preliminary knowledges and problem formulation

2.1. Motion equation of VLP

Consider a VLP shown in Fig. 1. Assume the rod is massless with a point mass sliding along the rod axis. Let $\theta(t)$ be the angle between the pendulum and the vertical, let $l(t)$ be the length of the pendulum from the O to the mass m, and let $f(t)$ be

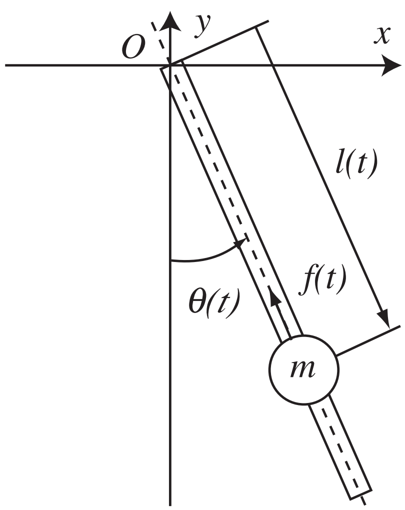

Fig. 1. Variable length pendulum (VLP).