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Grazing bifurcation in aeroelastic systems with freeplay nonlinearity

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ABSTRACT

A nonlinear analysis is performed to characterize the effects of a nonsmooth freeplay nonlinearity on the response of an aeroelastic system. This system consists of a plunging and pitching rigid airfoil supported by a linear spring in the plunge degree of freedom and a nonlinear spring in the pitch degree of freedom. The nonsmooth freeplay nonlinearity is associated with the pitch degree of freedom. The aerodynamic loads are modeled using the unsteady formulation. Linear analysis is first performed to determine the coupled damping and frequencies and the associated linear flutter speed. Then, a nonlinear analysis is performed to determine the effects of the size of the freeplay gap on the response of the aeroelastic system. To this end, two different sizes are considered. The results show that, for both considered freeplay gaps, there are two different transitions or sudden jumps in the system's response when varying the freestream velocity (below linear flutter speed) with the appearance and disappearance of quadratic nonlinearity induced by discontinuity. It is demonstrated that these sudden transitions are associated with a tangential contact between the trajectory and the freeplay boundaries (grazing bifurcation). At the first transition, it is demonstrated that increasing the freestream velocity is accompanied by the appearance of a superharmonic frequency of order 2 of the main oscillating frequency. At the second transition, the results show that an increase in the freestream velocity is followed by the disappearance of the superharmonic frequency of order 2 and a return to a simple periodic response (main oscillating frequency).

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1. Introduction

Concentrated nonlinearities, such as cubic stiffness and freeplay are commonly found in aeroelastic systems. Freeplay nonlinearities are exhibited by control surface attachments and are due to loosened mechanical linkages and manufacturing tolerances. The presence of freeplay nonlinearity in an aeroelastic system may lead to complex and undesirable responses, such as instabilities, limit cycle oscillations (LCO), chaos and abrupt bifurcations. Characterizing and understanding these undesirable behaviors has been the topic of many investigations. Virgin et al. [1], Conner et al. [2], Trickey et al. [3], and Vasconcellos et al. [4] have evaluated numerically and experimentally the effects of a freeplay nonlinearity in the flap degree of freedom on the response of an aeroelastic system. They showed that transitions from damped to periodic LCOs to quasi-periodic responses, and then, to chaotic motions can occur. These transitions were observed at airspeeds lower than the linear flutter speed.

The grazing bifurcation of limit cycles is a common discontinuity-induced bifurcations (DIBs) [5–8]. This bifurcation is caused when a periodic orbit reaches a boundary tangentially and, as such, can occur only in discontinuous systems such

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as shock sensors, gears, cutting tools, tapping mode atomic force microscopy, and aeroelastic systems with freeplay nonlinearities. Several investigations identified grazing bifurcations in different elastic structures undergoing impacts [9–14]. For this bifurcation, a special phenomenon arises during zero-speed incidence which is refereed to "grazing impacts". This phenomenon was first described by Whiston [12]. Grazing bifurcations have also been found in structural systems, such as spring-mass systems [10,13,15,14,16–18] and cantilever beams [9,11,19–22]. Luo and Gegg [23–25] presented an extensive investigation on sliding and grazing bifurcations in forced oscillators with dry friction. In these and other studies, the role of different parameters and excitations sources, such as low-speed impacts [15], friction and hard impacts [14], harmonic and aharmonic impacts [26], and off-resonance excitations [21] in the generation of grazing bifurcations were investigated.

Because freeplay nonlinearity is a nonsmooth and generally exists in aeroelastic systems, one would expect grazing bifurcations to take place resulting in specific responses of aeroelastic systems. In this work, grazing bifurcation in a two degrees of freedom aeroelastic system with a freeplay nonlinearity in the pitch degree of freedom is investigated and characterized. This system consists of a plunging and pitching rigid airfoil supported by linear spring in the plunge degree of freedom and a nonlinear spring in the pitch degree of freedom. The governing equations of the considered aeroelastic system and the nonsmooth function representing the freeplay nonlinearity are presented in Section 2. In Section 3, the aerodynamic loads are modeled based on the unsteady formulation. Linear and nonlinear analyses are performed in Sections 4 and 5, respectively. Summary and conclusions are presented in Section 6.

2. Nonlinear aeroelastic model

The aeroelastic system consists of a two-dimensional airfoil that has two degrees of freedom, namely the pitch and plunge motions, as illustrated in Fig. 1. These motions are denoted by h and α, respectively, and are measured at the elastic axis. The distance from the elastic axis to mid-chord is given by ab where a is a constant and b is the semi-chord length of the entire airfoil section. The mass center of the entire airfoil is located at a distance $x_\alpha b$ from the elastic axis. The stiffness of the springs of the plunge and pitch motions are denoted by k_h and k_α, respectively. The viscous damping forces are described through the coefficients c_h and c_α for plunge and pitch, respectively. Finally, U is used to denote the freestream velocity.

Using Lagrange's equations, the equations of motion governing this system are written as [27–30]:

$$\begin{bmatrix} m_T & m_w x_\alpha b \\ m_w x_\alpha b & I_\alpha \end{bmatrix} \begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha F(\alpha)/\alpha \end{bmatrix} \begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix}, \quad (1)$$

where m_T is the mass of the entire system (wing and support), m_w is the wing mass alone, I_α is the mass moment of inertia about the elastic axis, L and M are the aerodynamic lift and moment about the elastic axis, and $F(\alpha)$ is a function used to represent the freeplay nonlinearity associated with the pitch motion.

The values of the structural parameters of the aeroelastic system considered in this work are presented in Table 1. The function assigned to represent the discontinuous freeplay effects, $F(\alpha)$ is shown in Fig. 2 and given by:

$$F(\alpha) = \begin{cases} \alpha + \delta, & \text{if } \alpha < -\delta, \\ 0, & \text{if } |\alpha| \leqslant \delta, \\ \alpha - \delta, & \text{if } \alpha > \delta. \end{cases} \quad (2)$$

Different methods can be used to solve the governing equations, such as Henon's method which is applied in this work [31].

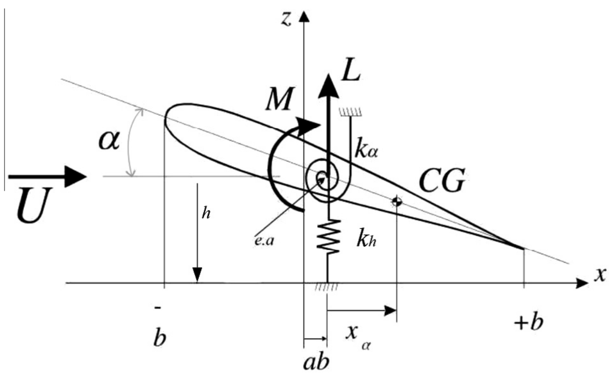

Fig. 1. Schematic of an aeroelastic system under uniform airflow.