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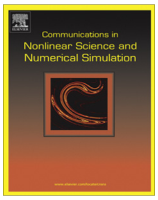

Buckling and stability analysis of a piezoelectric viscoelastic nanobeam subjected to van der Waals forces

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ABSTRACT

A study on the buckling and dynamic stability of a piezoelectric viscoelastic nanobeam subjected to van der Waals forces is performed in this research. The static and dynamic governing equations of the nanobeam are established with Galerkin method and under Euler–Bernoulli hypothesis. The buckling, post-buckling and nonlinear dynamic stability character of the nanobeam is presented. The quasi-elastic method, Leibnitz's rule, Runge–Kutta method and the incremental harmonic balanced method are employed for obtaining the buckling voltage, post-buckling characteristics and the boundaries of the principal instability region of the dynamic system. Effects of the electrostatic load, van der Waals force, creep quantity, inner damping, geometric nonlinearity and other factors on the post-buckling and the principal region of instability are investigated.

1. Introduction

Nanoelectromechanical systems (NEMS) are MEMS scaled to submicron dimensions [1]. In this size regime, they come with extremely high fundamental resonance frequencies, diminished active masses, and tolerable force constants [2], which will provide a revolution in applications such as sensors [3], medical diagnostics [4], nanomechanical oscillators [5], and data storage [6]. The initial research in science and technology related to nanoelectromechanical systems has therefore been taking place throughout the world in recent years. For example, Dequesnes and Tang [7] analyzed the static and dynamic behaviors of carbon nanotube-based switches based on the continuum mechanics and molecular dynamics method. Conley and Raman [8] explored the nonlinear and nonplanar dynamics of suspended nanotube and nanowire resonator, which could suddenly transition from a planar motion to whirling. Ansaril and Sahmani [9] presented different beam theories to analyze bending and buckling responses of nanobeams including surface stress effects. Gheshlaghi and Hasheminejad [10] analyzed the surface effect on nonlinear free vibration of nanobeams. Yang and Jia [11] studied the Pull-in instability of nano-switches using nonlocal elasticity theory. Sudak [12] studied the buckling of multiwalled carbon nanobute based on the Euler–Bernoulli beam model and the Eringen nonlocal theory, The van der Waals interactions between adjacent tubes were considered in Sudak's model. Xiang and Wang [13] analyzed the dynamic instability of cantilevered nanorods/nanotubes subjected to an end follower force by Eringen's nonlocal elasticity theory. Zhou and Wang [14] studied the nonlinear resonance of a piezoelectric nanowire.

In spite of substantial achievements in this field, an efficient, integrated, and customizable technique for actively driving and tuning NEMS resonators has remained elusive. The actuation method based on piezoelectric effect provides a means of

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Nomenclatures

l	the length of the piezoelectric viscoelastic nanobeam
b	the width of the piezoelectric viscoelastic nanobeam
h	the thickness of the piezoelectric viscoelastic nanobeam
g	the initial gap between the nanobeam and the substrates
$\varepsilon(x, z, t)$	the strain at an arbitrary point of the nanobeam along the x-direction
$\sigma(x, z, t)$	the stress at an arbitrary point of the nanobeam along the x-direction
d_{31}	the piezoelectric strain constant of the nanobeam
$E(t)$	the relaxation modulus of the nanobeam
E_V	the electric field intensity
E_0	the initial Young's modulus of the nanobeam
E_1	the durable modulus
E_2	the creep modulus
λ	the coefficient of the element relaxation
v	the Poisson ratio
$w(x, t)$	transverse displacement at an arbitrary point of the nanobeam in the x-direction
κ	the curvature of the neutral axis of the deformed nanobeam
u_0	the displacement of the corresponding point in the neutral axis along the x-direction
$N(x, t)$	the axial force of the nanobeam in the x-direction
$M(x, t)$	the bending moment of the nanobeam in the x-direction
I	the cross-section inertia moment of the nanobeam
S	the area of cross-section of the nanobeam
H	the Hamaker constant
F_w	the van der Waals force per unit length of the nanobeam
V	the amplitude of the AC voltage in the nanobeam
θ	half of the frequency of the AC voltage in the nanobeam
ρ	the mass density of the nanobeam

directly converting an electric field into the mechanical strain [15]. However, piezoelectric technology scaled to nanoscale raises a new challenge, it is hard to deposit ultrathin films that retain the properties of their thicker microscale counterparts [16], and consequently the potential of the ultrathin films applications will be limited. Therefore, studying the electromechanical characteristic of piezoelectric structure is necessary for the application of piezoelectric nanostructure. Sadek and Masmanidis [15,17] have studied the vibrational character of piezoelectric NEMS composed of GaAs nanofilm or GaAs nanowire. In their models, the resonance frequency was controlled by bias voltage (it is called piezoelectric voltage in this paper) which could be DC and AC signals, but the maximum piezoelectric voltage that can be applied was not given. On the other hand, most of the models presented in the research above were established without considering the viscoelasticity, while in the literatures it has revealed that some materials used to manufacture micro/nanostructures, such as the piezoelectric material GaAs mentioned in [14,16,18], also exhibit viscoelasticity [19]. As a result, these theoretical calculations and simulations about NEMS without considering viscoelasticity may lead to a decrease of accuracy. At nanoscale, the additional effect of van der Waals force should be considered [20,21]. Applying piezoelectric voltage on GaAs nanobeam is equivalent to implementing an axial stress to the nanobeam. Thus based on Euler beam theory, when piezoelectric voltage reaches a certain value, the buckling and post-buckling phenomena will occur in the nanobeam structure. If piezoelectric voltage (bias voltage) is AC voltage and its amplitude is large enough, the dynamic instability will occur in the system, too. Hence, discussing the buckling, post-buckling and the dynamic instability phenomena of NEMS or MEMS, containing piezoelectric viscoelastic component subjected to van der Waals force, can provide more precise guidance to avoid the failure of these NEMS or apply the physical properties of these phenomena in NEMS.

The present research investigates the nonlinear dynamical characteristics of a nanobeam that couples both piezoelectricity and viscoelasticity. A novel model is established for studying the buckling, post-buckling and nonlinear dynamic stability of the simple–simple supported piezoelectric viscoelastic nanobeam subjected to van der Waals forces. Numerical simulations with employment of the model are conducted via Runge–Kutta method and the incremental harmonic balance method (IHBM). The effects of related system parameters on the buckling, post-buckling and the principal region of instability are also studied and presented numerically and graphically.

2. Piezoelectric viscoelastic nanobeam model development

Consider a simple–simple supported, piezoelectric viscoelastic nanobeam under a periodic electrostatic load between two substrates. The two dimensional mechanical sketch of the piezoelectric viscoelastic nanobeam is shown as Fig. 1, where the nanobeam has length l, width b and thickness h, and g is the initial gap.