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Short communication

A nonlinear model to generate the winner-take-all competition

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ABSTRACT

This paper is concerned with the phenomenon of winner-take-all competition. In this paper, we propose a continuous-time dynamic model, which is described by an ordinary differential equation and is able to produce the winner-take-all competition by taking advantage of selective positive–negative feedback. The global convergence is proven analytically and the convergence rate is also discussed. Simulations are conducted in the static competition and the dynamic competition scenarios. Both theoretical and numerical results validate the effectiveness of the dynamic equation in describing the nonlinear phenomena of winner-take-all competition.

1. Introduction

Competition widely exists in nature and the society. Among different kinds of competitions, winner-take-all competition refers to the phenomena that individuals in a group compete with each others for activation and only the one with the highest input stays activated while all the others deactivated. Examples of this type of competition include the dominant growth of the central stem over others [1], the contrast gain in the visual systems through a winner-take-all competition among neurons [2], competitive decision making in the cortex [3,4], cell fate competition [5,6], etc.

Although many phenomena, as exemplified above, demonstrate the same winner-take-all competition, they may have different underlying principles in charge of the dynamic evolution. There are various mathematic models presented to describe this type of competition phenomena, e.g., the N species Lotka–Volterra model [7,8], interactively spiking FitzHugh–Nagumo Model [9–11], optimization based model [12,13], discrete-time different equation model [14], neural network model [15,16], lateral inhabitation model [17,18], to name a few. However, these models are often very complicated due to the compromise with experimental realities in the particular fields. Consequently, the essence of the winner-take-all competition may be embedded in the interaction dynamics of those models, but difficult to tell from the sophisticated dynamic equations. Motivated by this, a simple ordinary differential equation model with a direct and intuitive explanation is presented in this paper and it is expected to cast lights to researchers on the principle of competition phenomena in different fields.

The remainder of this paper is organized as follows: in Section 2, the analytical model is presented and the underlying competition mechanism is explained from a selective positive–negative feedback perspective. In Section 3, the competition

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behavior and the convergence results are proven rigourously by means of nonlinear stability tools. In Section 4, illustrative examples are given to show the effectiveness of the proposed model. The paper is concluded in Section 5.

2. The model

The proposed model has the following dynamic for the ith agent in a group of totally n agents,

$$\dot{x}_i = c_0(u_i - \|x\|^2)x_i \tag{1}$$

where $x_i \in \mathbb{R}$ denotes the state of the i agent, $u_i \in \mathbb{R}$ is the input and $u_i \geqslant 0$, $u_i \neq u_j$ for $i \neq j$, $\|x\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$ denotes the Euclidean norm of the state vector $x = [x_1, x_2, \ldots, x_n]^T$, $c_0 \in \mathbb{R}$ $c_0 \geqslant 0$ is a scaling factor.

The dynamic Eq. (1) can be written into the following compact form by stacking up the state for all agents,

$$\dot{x} = c_0(u \circ x - \|x\|^2 x) \tag{2}$$

where $x = [x_1, x_2, \ldots, x_n]^T$, $u = [u_1, u_2, \ldots, u_n]^T$, the operator '$\circ$' represents the multiplication in component-wise, i.e., $u \circ x = [u_1x_1, u_2x_1, \ldots, u_nx_n]^T$.

Remark 1. In the dynamic Eq. (1), all quantities on the right hand side can be obtained locally from the ith agent itself (u_i and x_i) except the quantity $\|x\|^2$, which reflects the effort from other agents over the ith one (as sketched in Fig. 1). Actually, $\|x\|^2 = x_1^2 + x_2^2 + \cdots + x_n^2$ is the second moment about the origin of the group of agents and it is a statistic of the whole group. In this regard, the dynamic model (1) implies that the winner-take-all competition between agents may emerge in a multi-agent system if each agent accesses the global statistic $\|x\|^2$ (instead of exactly knowing states of all the other agents) besides its own information.

As will be stringently demonstrated later, the agent with the largest input will finally win the competition and keep active while all the other agents will be deactivated to zero eventually. Before proving this result rigorously, we present a intuitive explanation of the result in a sense of positive feedback vs. negative feedback. Note that the term $c_0u_ix_i$ in Eq. (1) provides a positive feedback to the state variable x_i as $u_i \geqslant 0$ while the term $-c_0\|x\|^2x_i$ supplies a negative feedback. For the ith agent, if $u_i = \|x\|^2, x_i$ will keep the value. If $u_i < \|x\|^2$, the positive feedback is less than the negative feedback in value and the state value attenuates to zero. In contrast, if $u_i > \|x\|^2$, the positive feedback is greater than the negative feedback and the state value tends to increase as large as possible until the resulting increase of $\|x_i\|$ surpasses u_i. Particularly for the winner, say the k^*th agent, $u_{k^*} > u_i$ holds for all $i \neq k^*$. In this case, all agents have negative feedbacks and keep reducing in values until $\|x\|^2$ reduces to the value of u_k when $u_k < \|x\|^2$. Otherwise when u_k is slightly greater than $\|x\|^2$ (by slightly greater we mean $u_k > \|x\|^2 > u_l$ with l denoting the agent with the second largest state value), only the winner has a positive feedback and has an increase in its state value while all the other agents have negative feedbacks and keep reducing until $\|x\|^2$ equals u_k. Under this selective positive–negative feedback mechanism, the winner finally stays active at the value $u_{k^*} = \|x\|^2$ while the losers are deactivated to zero.

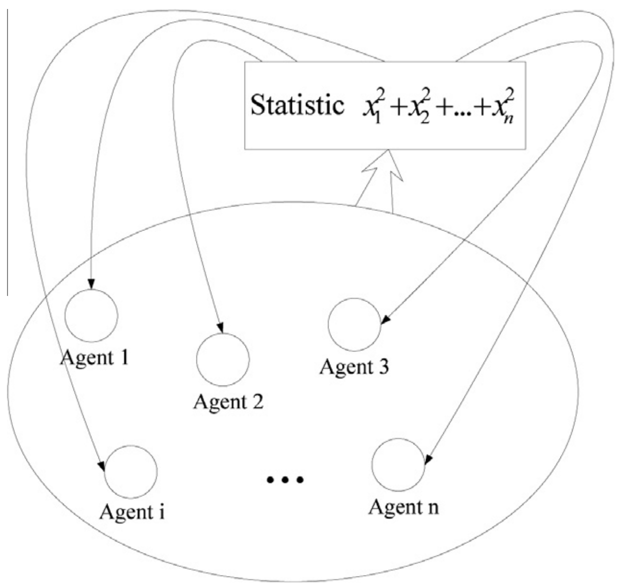

Fig. 1. Information flow for the agent dynamics.