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Investigation of micropolar fluid flow in a helical pipe via asymptotic analysis

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ABSTRACT

In this paper we study the flow of incompressible micropolar fluid through a pipe with helical shape. Pipe's thickness and the helix step are considered as the small parameter ε. Using asymptotic analysis with respect to ε, the asymptotic approximation is built showing explicitly the effects of fluid microstructure and pipe's distortion on the velocity distribution. The error estimate for the approximation is proved rigorously justifying the obtained model.

1. Introduction

Engineering practice requires extensive knowledge of flow through curved pipes. Helically coiled pipes are well known types of curved pipes which offer several advantages over straight pipes due to their compactness i.e. increased surface of the pipe within the same volume. Therefore, they have been used in wide variety of applications such as air conditioners, refrigeration systems, central heating radiators, chemical reactors etc. In this paper we study the flow through a helical pipe (see Fig. 1) whose central curve is parameterized by

$$x_1 \mapsto x_1\mathbf{i} + a\cos\frac{x_1}{\varepsilon}\mathbf{j} + a\sin\frac{x_1}{\varepsilon}\mathbf{k}, \quad x_1 \in [0,\ell],$$

with small parameter ε being the ratio between pipe's thickness and its length. It means that the distance between two coils of the helix (*helix step*) and pipe's thickness have the same small order $\mathcal{O}(\varepsilon)$, while the diameter of the helix is larger, of order $\mathcal{O}(1)$. Such assumptions cover a large variety of realistic coiled pipes which can be found in many devices.

We suppose that the pipe is filled with incompressible micropolar fluid. Among various non-Newtonian fluid models, the model of micropolar fluid have gained much attention since it captures the effects of local structure and micro-motions of the fluid elements which cannot be described by the classical models. Physically, it represents fluids consisting of a large number of small spherical particles uniformly dispersed in a viscous medium. Assuming that the particles are rigid and ignoring their deformation, the adequate theory was proposed by Eringen [1] in '60s. The related mathematical model is based on the introduction of new vector field, the angular velocity field of rotation of particles (*microrotation*), to the classical pressure and velocity fields. Correspondingly, one new (vector) equation is added, expressing the conservation of the angular momentum. As a result, a significant generalization of the Navier–Stokes equations is obtained with four new viscosities introduced. Micropolar fluid flow has been extensively studied due to its practical importance in various applications. Some recent results can be found in papers by Dupuy et al. [2,3], Abdullah and Amin [4], Ishaka et al. [5] and Pažanin [6,7]. We also

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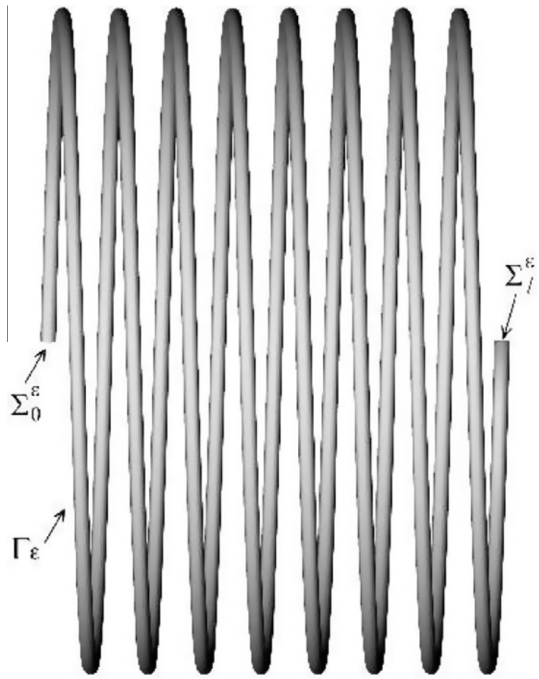

Fig. 1. Helical pipe.

emphasize the monograph by Lukaszewicz [8] which provides a unified picture of the mathematical theory underlying the applications of micropolar fluids.

The goal of this paper is to rigorously derive the asymptotic model with high order of accuracy explicitly acknowledging the effects of fluid microstructure and pipe's distortion. Since the governing problem is described by nonlinear coupled system of PDEs (see (3)–(5)) posed in the domain with complex geometry, it is impossible to derive the exact solution. For that reason, singular perturbation technique is employed. The main idea is to introduce the curvilinear coordinates attached to the Frenet basis of the helix and, after writing the micropolar equations in the appropriate basis, to apply two-scale asymptotic expansion in powers of the small parameter ε. We manage to compute explicitly higher-order correctors in the velocity expansion clearly showing the effects we sought for. Moreover, we evaluate the difference between the original solution and formally derived asymptotic approximation and prove satisfactory error estimates in the appropriate norm. By taking into account the effects of the fluid microstructure we believe that the obtained model can upgrade the numerical simulations of helical pipe flows and improve the known engineering practice.

We conclude this section by giving some bibliographic remarks. There exists a large number of papers dealing with the flow characteristics of classical, Newtonian fluid in helical pipes. Let us mention only those that influenced our work. Wang [9] proposed a non-orthogonal coordinate system to investigate the effects of curvature and torsion on the low-Reynolds number flow through a helical pipe. The introduction of an orthogonal system of coordinates along a spatial curve allowed Germano [10,11] to explore in a simpler way the influence of pipe's geometry on the effective flow. Yamamoto et al. [12] studied experimentally the effects of pipe's distortion on the flow characteristics. Numerical simulations of helical pipe flows can be found in Liu and Masliyah [13] and Wang and Andrews [14]. Rigorous justification of the formally derived asymptotic model describing the Newtonian flow through a helically coiled pipe was done by Marušić-Paloka and Pažanin [15].

Recently, in paper by Pažanin [16], micropolar fluid flow has been addressed in the case of general central curve with flexion κ and torsion τ of order $\mathcal{O}(1)$. In the present paper we treat the problem in which $\kappa = \frac{a}{a^2+\varepsilon^2} = \mathcal{O}(1)$ but $\tau = \frac{\varepsilon}{a^2+\varepsilon^2} = \mathcal{O}(\varepsilon)$ so this particular case does not enter in the framework from [16]. We find this setting to be more challenging from the point of view of asymptotic analysis due to technical difficulties caused by the specific pipe's geometry.[1]

2. Setting of the problem

2.1. The geometry

Let $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ denotes the standard Cartesian basis. We consider the helix given by the parametrization

[1] In this setting, Frenet basis depends on small parameter ε (it follows the pipe's coils and thus oscillates with period ε) and various precautions in that direction should be done.