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A new lattice model of two-lane traffic flow with the consideration of optimal current difference

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ABSTRACT

In this paper, a new lattice model of traffic flow is proposed with the consideration of the optimal current difference for two-lane system. The linear stability condition is derived through linear stability analysis, which shows that the optimal current difference term can improve the stability of traffic flow. The mKdV equation is obtained through nonlinear analysis. Thus the space of traffic flow is divided into three regions: the stable region, the metastable region and the unstable region respectively. Moreover, numerical simulation confirms that the traffic jam can be suppressed efficiently by considering the optimal current difference effect in extended lattice model of two-lane traffic flow.

1. Introduction

Traffic jam has attracted more and more attention of many scholars since traffic problem becomes more complex with the rapid increase of traffic flux. Then, many traffic models have been put forward to investigate traffic phenomena. Especially, the lattice model firstly proposed by Nagatani [1,2] is a charming hydrodynamic model to analyze the jamming transition evolution of traffic flow. Subsequently, lots of extended lattice models [3–22] have been developed by taking different factors into account. However, above models are only suited to describe some traffic phenomena in single lane. In real traffic, multiple lanes are the usual form of traffic. Hereafter, Nagatani [23] further presented a two-lane lattice model of traffic flow. Thereafter, some two-lane lattice models [24,25] were derived with lane changing behaviors. However, these models did not consider the optimal current difference which may have important influence on traffic flow in two-lane system. In this paper, a new lattice model is proposed to enhance the stability of traffic flow with the consideration of optimal current difference in two-lane system. The linear stability analysis and nonlinear analysis are carried out to investigate the effect of optimal current difference on the traffic stability and jamming transition.

2. The new model

Fig. 1 shows the schematic model of traffic flow on a two-lane highway [23]. Lane changing rate $\gamma|\rho_0^2 V'(\rho_0)|(\rho_{2,j-1} - \rho_{1,j})$ was supposed from lane 2 to lane 1 if the density at site $j-1$ on lane 2 is higher than that at site j on lane 1. where, γ denotes the rate constant coefficient with dimensionless. Similarly, lane changing rate $\gamma|\rho_0^2 V'(\rho_0)|(\rho_{1,j} - \rho_{2,j+1})$ was assumed from lane 1 to lane 2 if the density at site j on lane 1 is higher than that at site $j+1$. Thus, the continuity equations on two-lane highway proposed by Nagatani were formulated as follows [23]:

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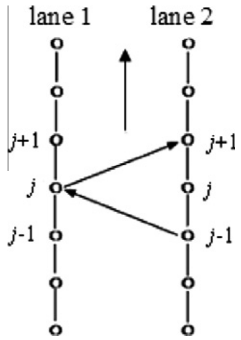

Fig. 1. The schematic model of traffic flow on a two-lane highway.

$$\partial_t \rho_{1,j} + \rho_0(\rho_{1,j} v_{1,j} - \rho_{1,j-1} v_{1,j-1}) = \gamma|\rho_0^2 V'(\rho_0)|(\rho_{2,j+1} - 2\rho_{1,j} + \rho_{2,j-1}) \tag{1}$$

$$\partial_t \rho_{2,j} + \rho_0(\rho_{2,j} v_{2,j} - \rho_{2,j-1} v_{2,j-1}) = \gamma|\rho_0^2 V'(\rho_0)|(\rho_{1,j+1} - 2\rho_{2,j} + \rho_{1,j-1}) \tag{2}$$

By adding Eqs. (1) and (2), one can obtain the following continuity equation:

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = \gamma|\rho_0^2 V'(\rho_0)|(\rho_{j+1} - 2\rho_j + \rho_{j-1}) \tag{3}$$

where ρ_0, ρ and v mean the average density, the local density and local velocity respectively. $\rho_j = (\rho_{1,j} + \rho_{2,j})/2$, $\rho_j v_j = (\rho_{1,j} v_{1,j} + \rho_{2,j} v_{2,j})/2$ and $V_e(\rho_j) = (V(\rho_{1,j}) + V(\rho_{2,j}))/2$. The following evolution equation of the two-lane traffic was incorporated [23]:

$$\rho_j(t+\tau) v_j(t+\tau) = \rho_0 V(\rho_{j+1}) \tag{4}$$

However, Nagatani's lattice model didn't consider the optimal current difference on two-lane highway. Therefore, based on Nagatani's two-lane lattice model [23], we present a novel evolution equation with the consideration of the optimal current difference effect as follows:

$$\rho_j(t+\tau) v_j(t+\tau) = \rho_0 V(\rho_{j+1}) + \lambda(\rho_0 V(\rho_{j+2}) - \rho_0 V(\rho_{j+1})) \tag{5}$$

where, $\Delta F_{j+1} = \rho_0 V(\rho_{j+2}) - \rho_0 V(\rho_{j+1})$, which denotes optimal current difference on site $j+1$ at time t; λ is the reaction coefficient of optimal current difference. The optimal velocity function $V(\rho)$ is adopted as follows [1,2]:

$$V(\rho) = (v_{\max}/2)[\tanh(1/\rho - h_c) + \tanh(h_c) \tag{6}$$

where, h_c and $v_{\max}$ represent the safety distance and the maximal velocity. By eliminating the speed v in Eqs. (3) and (5), one obtains the density equation as follows:

$$\begin{aligned}&\rho_j(t+2\tau) - \rho_j(t+\tau) + \tau\rho_0^2\Big[V(\rho_{j+1}) - V(\rho_j)\Big] + \lambda\tau\rho_0(\Delta F_{j+1} - \Delta F_j)\\ &\quad - \tau\gamma|\rho_0^2 V'(\rho_0)|(\rho_{j+1}(t+\tau) - 2\rho_j(t+\tau) + \rho_{j-1}(t+\tau)) = 0\end{aligned} \tag{7}$$

3. Linear stability analysis

A constant density ρ_0 and optimal velocity $V(\rho_0)$ are defined for the steady state of the uniform traffic flow on two-lane highway. y_j is supposed a small deviation from the steady-state flow on site j.

$$\rho_j(t) = \rho_0, \quad \rho_j(t) = \rho_0 + y_j \tag{8}$$

By substituting Eq. (8) into Eq. (7) and linearizing it, one obtains

$$\begin{aligned}&y_j(t+2\tau) - y_j(t+\tau) + \rho_o^2 V'(\rho_0)\Delta y_j(t) + \lambda\rho_o^2 V'(\rho_0)(\Delta y_{j+1}(t) - \Delta y_j(t))\\ &\quad - \gamma|\rho_0^2 V'(\rho_0)|(y_{j+1}(t+\tau) - 2y_j(t+\tau) + y_{j-1}(t+\tau)) = 0\end{aligned} \tag{9}$$

where, $\Delta y_j = y_{j+1} - y_j$ and $V'(\rho_0) = dV(\rho)/d\rho|_{\rho=\rho_0}$.

By expanding $y_j = A\exp(ikj + zt)$, one derives the following equation of z:

$$e^{2z\tau} - e^{z\tau} + \tau\rho_o^2 V'(\rho_0)(e^{ik} - 1) + \lambda\tau\rho_o^2 V'(\rho_0)(e^{2ik} - e^{ik} + 1) - \tau\gamma|\rho_0^2 V'(\rho_0)|e^{z\tau}(e^{ik} - 2 + e^{-ik}) = 0 \tag{10}$$

Inserting $z = z_1(ik) + z_2(ik)^2 + \cdots$ into Eq. (10) and neglecting the higher order terms, one can obtain the first-order and second-order terms of ik as follows: