Commun Nonlinear Sci Numer Simulat 16 (2011) 4745–4755

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Phase trajectory portrait of the vibro-impact forced dynamics of two heavy mass particles motions along rough circle

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ARTICLE INFO

Article history:
Available online 1 July 2011

Keywords:
Two heavy mass particles
Rough circle
Friction
Vibro-impact
Phase trajectory branches
Singular points
Bifurcation of equilibrium position
Trigger of coupled one side singularities
Numerical experiment
One frequency force

ABSTRACT

Forced vibro-impact dynamics of the two heavy mass particle motions, in vertical plane, along rough circle with Coulomb's type friction and one, one side impact limiter is considered in combinations of applied analytical and numerical methods. System of two differential double equations, each for one of two heavy mass particle motions along same rough circle are composed with corresponding initial conditions as well as impact conditions. By use software package tools differential double equations are numerically integrated for obtaining phase portrait of phase trajectory branches for different mass particles initial kinetic states. By series of the phase trajectory branches for each mass particle motion between two impacts or between impact and alternation of the Coulomb's friction force direction, two phase trajectory graphs of the system vibro-impact non-linear dynamics are composed. Different software tools are used as helping tools for calculate time moments of the series of the impacts between mass particles, as well as positions of the impacts, necessary for calculations of the impact velocities of the mass particles before and after impacts. Some comparison between forced and free vibro-impact dynamics of the two heavy mass particles in vertical plane, along rough circle with Coulomb's type friction and one, one side impact limiter is done. Trigger of coupled one side singularities in phase portraits are identified.

1. Introduction

Non-linear phenomena in dynamics of vibro-impact systems are special types of non-linearity caused by basic system non-linearity coupled with series of impacts, followed by kinetic parameters discontinuities and alternations of the motion and velocity direction, as strong non-linearity. Also, no ideal constraints with Coulomb's type friction forces are source of the strong non-linearity caused by alternations of the friction force directions. Nowadays, research interest for investigation of this type non-linear phenomena increase exponentially. Theoretical knowledge and analytical results concerning vibro-impact system dynamics are very valuable in the trends that large numbers of researchers focused to the computation investigations. Also, knowledge of vibro-impact phenomena and vibro-dynamics with impacts is very important for engineering applications, taking into account that working processes of many new engineering system are based on the vibro-impact processes. Series of monographs by Babickii and Kolovskii [1,3], Babickii [2] and Peterka [21] and papers by Bapat and Popplewell [4] and Peterka [22–24] and other, contain important scientific and advances to the topic properties of vibro-impact dynamics with corresponding particular methodology applied to the particular classes of the vibro-impact system dynamics.

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doi:10.1016/j.cnsns.2011.05.027

Some classical problems of mechanical system motion with no ideal constraints and friction as well as an oscillator with Coulomb's type friction are presented in the university books on the level of monographs written by Rašković [26,25] contain basic analytical results in this topic. Exellant papers written by Matrosov and Finogenko [19,20] contain the theory of right solutions of equations for mechanical systems dynamics with sliding friction in one-degree-of-freedom kinematics pairs, which has been developed by the authors. Also, some difficulties bound up with "non-uniqueness" of motion in course of description of such systems, which are known as Painlevé's paradoxes are discussed.

New series of published papers by Hedrih (Stevanović) [8,10,11,13] presented new research results regarding heavy mass particle motions along circles which rotate, as well as hybrid dynamics in the form of the coupled rotations.

Analysis of the mathematical pendulum dynamics in the field with turbulent damping (see Ref. Stoker [27]) and papers written by Hedrih (Stevanović) [9,6,5,7,12](2006,2007) related to the heavy mass particle dynamics along rotate circle as well as to the heavy mass particle dynamics along rough curvilinear line with Amntons–Coulomb's type frictions are basic inspiration of the series of research results of vibro-impact non-linear dynamics co-authored by Hedrih et al. (see Refs. [14–18]) and presented in the published co-author papers in period 2009 and 2010 and listed in the reference list of this paper, as well as in the magistar of science thesis defended by Jović 2009.

Also, new results included and presented in this our paper was obtained in continuation of our research interest in the topic of vibro-impact dynamics of the systems with Coulomb's type friction and based on the basis previous listed and published our results.

2. Differential double equations of vibro-impact dynamics of two heavy mass particle forced motion along rugh circle

Let consider forced vibro-impact dynamics of the two heavy mass particle motions, in vertical plane, along rough circle with Coulomb's type friction and one, one side impact limiter of the angular elongations of the right hand side heavy mass particle. System is shown in Fig. 1a*. A detail of the system is visible in Fig. 1b* with plan of the active and reactive forces.

System under the consideration have two degree of freedom on the basis of the forced oscillations two heavy mass particles with masses m_1 and m_2. Heavy mass particle are under the action of the two external eccitations, applied in tangential directions to the circle line and each one-frequency in the following forms: $F_1(t) = F_{\Omega_1} = F_{10}\cos\Omega_1 t$ and $F_2(t) = F_{\Omega_2} = F_{20}\cos\Omega_2 t$, where F_{10} and F_{20} are amplitudes of the external forces, and Ω_1 and Ω_2 are frequencies of the corresponding external forces. For independent generalized coordinates we use angular coordinate φ_1 and φ_2 of the mass particle arbitrary positions on the rough circle, starting of the equilibrium (nul) positions at corresponding ideal circle in vertical plane. At initial moment, corresponding initial starting positions and angular velocities of the heavy mass particles are: initial positions φ_{10} and φ_{20} with initial starting angular velocities $\dot{\varphi}_{10}$ and $\dot{\varphi}_{20}$. Position of the one side impact limiter is determined by angular coordinate $\varphi_{ul} = \delta$.

2.1. Differential double equations of forced no impact motion of heavy mass particles

For begining let consider phase trajectory in the phase plane of the no impact dynamics of the heavy mass particle forced motion along rough circle in vertical plane excited by one frequency external forces with corresponding initial conditions of

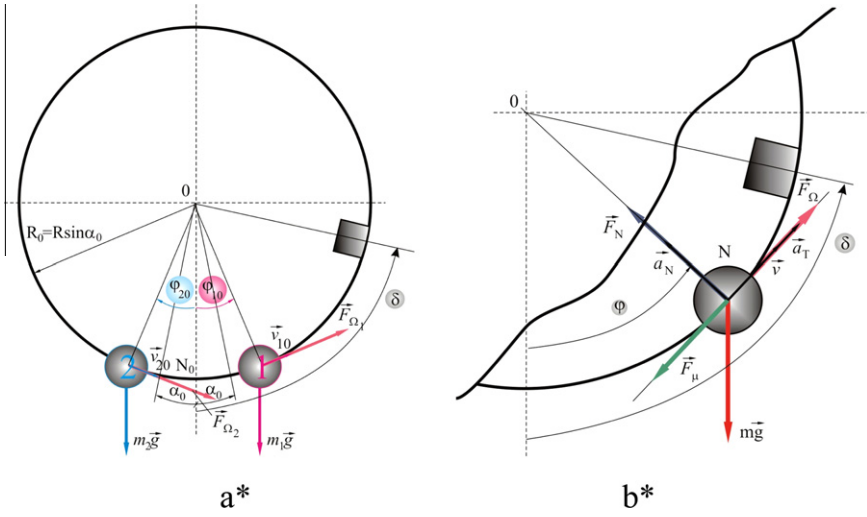

Fig. 1. Forced vibro-impact dynamics of the two heavy mass particle motions, in vertical plane, along rough circle with Coulomb's type friction and one, one side impact limiter of the angular elongations of the right hand side heavy mass particle. (a*) Initial positions of the mass particle on the rough circle in vertical plane. (b*) Arbitrary position of a heavy mass particle at moment t on the rough circle with plan of the active and reactive forces.