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Simultaneous effects of partial slip and thermal-diffusion and diffusion-thermo on steady MHD convective flow due to a rotating disk

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ABSTRACT

The purpose of present research is to derive analytical expressions for the solution of steady MHD convective and slip flow due to a rotating disk. Viscous dissipation and Ohmic heating are taken into account. The nonlinear partial differential equations for MHD laminar flow of the homogeneous fluid are reduced to a system of five coupled ordinary differential equations by using similarity transformation. The derived solution is expressed in series of exponentially-decaying functions using homotopy analysis method (HAM). The convergence of the obtained series solutions is examined. Finally some figures are sketched to show the accuracy of the applied method and assessment of various slip parameter γ, magnetic field parameter M, Eckert Ec, Schmidt Sc and Soret Sr numbers on the profiles of the dimensionless velocity, temperature and concentration distributions. Validity of the obtained results are verified by the numerical results.

1. Introduction

It is established fact now that problems in engineering can be modeled by the ordinary/partial differential equations. The development of exact solutions of the nonlinear ordinary/partial differential equations is of great importance. The perturbation technique [1,2] is one of the analytical methods to solve the nonlinear differential equations. This technique is widely used by the engineers for the solution of practical problems. Interesting and important results have been obtained by using this technique. This technique requires small/large parameter in the governing equation. Hence it is necessary to search technique which does not require small/large parameter at all. Because of the nonlinearities in the reduced differential equation, no analytical solution is available and the nonlinear equation is usually solved numerically subject to boundary conditions, one of which is prescribed at infinity. Although with the advancement of the symbolic computation software such as MATHEMATICA, MAPLE, MATLAB and so on approximate analytic methods for nonlinear problems have been adopted by many researchers. Among these are homotopy perturbation method (HPM) [3], homotopy analysis method (HAM) [4] and the variational iteration method (VIM) [5]. Liao [4] has described an analytical technique in this direction which does not require small parameters and thus can be applied to solve nonlinear problems without having small/large parameter. This technique is based on homotopy [6], which is an important concept in topology. Further, HAM has the ability to adjust and control the convergence region of obtained solutions through the convergence control parameter h and an appropriate initial guess.

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Unlike perturbation methods, the HAM does not depend on any small physical parameters. Thus, it is valid for both weakly and strongly nonlinear problems. Besides, different from all other analytic methods, the HAM provides us a simple way to adjust and control the convergence region of the series solution by means of convergence control parameter.

The HAM has been already successfully applied to solve many types of nonlinear problems such as the flows of non-Newtonian fluids over a stretching sheet [7,8], unsteady three-dimensional flows [9], the steady three-dimensional problem of condensation film on inclined rotating disk [10], Jaulent–Miodek equations [11], Burgers and regularized long wave equations [12], two-dimensional viscous flow in a rectangular domain bounded by two moving porous walls [13], the nonlinear equations arising in heat transfer [14], linear and nonlinear fractional partial differential equations [15,16] and many other problems. All of these successful applications verified the validity, effectiveness and flexibility of the HAM.

The flow due to rotating disks has received much attention in the several industrial and engineering processes. The reason of such a great interest can be attributed to its having a three-dimensional exact similarity solution which is significant in the study of engineering flows in rotating machinery, centrifugal pumps, viscometers and some aerodynamic related problems in fluid mechanics. The pioneering study of fluid flow due to an infinite rotating disk has been carried by von Karman [17]. He first formulated the problem and then the governing partial differential equations have been reduced to the ordinary differential equations by defining an appropriate transformations. The influence of an external uniform magnetic field on the flow due to a rotating disk was studied in [18] without Hall effects. Attia and Aboul-Hassan [19] studied the steady hydromagnetic problem with the Hall effects. Later Attia [20] extended the problem for the effect of uniform suction or injection in the MHD flow due to a rotating disk when both Hall and ion-slip currents are present.

Thermodiffusion, also called thermal diffusion or Soret effect corresponds to species differentiation developing in an initial homogeneous mixture submitted to a thermal gradient. On the other hand, the Soret effect has been also utilized for isotope separation and in mixture between gases with very light molecular weight, such as H_2 or He, and of medium molecular weight, such as H_2 or air. In many studies Dufour and Soret effects have been neglected on the basis that these are of smaller order of magnitude than the effects described by Fourier's and Fick's laws. Eckert and Drake [21] present the several cases when the Dufour effect cannot be neglected. In view of the importance of above-mentioned effects, Kafoussias and Williams [22] studied thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary-layer flow with temperature dependent viscosity. Postelnicu [23] numerically discussed the influence of a magnetic field and heat and mass transfer through natural convection from the vertical surfaces in a porous media by considering Soret and Dufour effects. Recently, Maleque and Sattar [24] obtained the numerical solution of MHD free-convective and mass transfer flow over an infinite vertical porous plate with thermal-diffusion effects.

In several problems, thermo-diffusion (Soret) and diffusion-thermo (Dufour) effects can be negligible with respect to the effects described by Fourier's and Fick's laws. These effects are considered as second order phenomena and may become significant in areas such as hydrology, nuclear waste disposal, geothermal energy, etc. Anghel et al. [25] analyzed both the Soret and Dufour effects on free convection boundary layer over a vertical surface embedded in a porous medium and noticed an appreciable change in the flow field. The effect of magnetic field and double dispersion on free convective heat and mass transport considering the Soret and Dufour effects in a non-Darcy porous medium has been studied by Partha et al. [26]. With these studies one can conclude in some problems Soret and Dufour effects are very important. Numerical solutions of complicated MHD problems are investigated in [27,28].

The main goal of the present study is to find the analytical solution for flow of a homogeneous electrically conducting fluid due to an infinite rotating disk in the presence of heat and mass transfer. Analytic solutions for the velocity, temperature and the concentration distributions are obtained using a powerful, easy-to-use technique, namely the HAM. Here the series solution is firstly computed and then its convergence is properly discussed. Finally, the graphs are plotted and discussed for the variations of slip parameter γ, magnetic field parameter M, Eckert Ec, Schmidt Sc and Soret Sr numbers.

Therefore, this paper has been organized as follows. In Section 2, the flow analysis and mathematical formulation are presented. In Section 3, we extend the application of the HAM to construct the approximate solutions for the governing equations. The convergence of the HAM are discussed in section 4. Section 5 contains the results and discussion. The conclusions are summarized in Section 6.

2. Problem statement and mathematical formulation

We consider the problem of steady hydromagnetic convective and slip flow due to a rotating disk in the presence of viscous dissipation and Ohmic heating. The effects of thermo-diffusion (Soret effect), diffusion-thermo (Dufour effect) effects and heat and mass transfer are accounted. Fig. 1 shows the physical model and geometrical coordinates. The disk at $z = 0$ rotates with constant angular velocity Ω (where z is the vertical axis in the cylindrical coordinates system with r and Φ as the radial and tangential axes respectively). The components of the flow velocity are (u, v, w) in the directions of increasing (r, Φ, z), respectively, the pressure is P and the density of the fluid is ρ. T and C are the fluid temperature and concentration, respectively and the surface of the rotating disk is maintained at a uniform temperature T_w and uniform concentration C_w. Far away from the surface, the free stream is kept at a constant temperature, concentration, pressure, T_∞, C_∞ and P_∞, respectively. The viscous fluid is electrically conducting. The external uniform magnetic field is applied perpendicular to the surface