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# A velocity observer design for tracking task-based motions of unicycle type mobile robots

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ABSTRACT

The paper presents a design of a nonlinear velocity observer and its application within a model-based tracking control strategy for tracking task-based motions of unicycle type mobile robots. The strategy is the model reference tracking control strategy for programmed motion and it enables switching between controllers employed in it to improve a tracking precision as well as switching between coordinates used for modeling based on a type of a nonholonomic system. The strategy benefits by adding the velocity observer to its architecture due to the reduction of a number of measurements needed for feedback tracking.


## 1. Introduction

Mobile robots may be equipped with position and velocity measurement devices, however signals obtained this way can be noise sensitive. Velocity signals can be obtained by differentiation of position signals but when calculated for low or high velocities they can be inadequate. To reduce weight and costs, robots are often not equipped with velocity sensors. These limitations can be circumvented by a velocity observer design. The main challenge in designing an observer–controller feedback for nonholonomic systems is the presence of the Coriolis term in their dynamic control model, which results in quadratic cross terms of unmeasured velocities. For this reason observer designs proposed for manipulators cannot be directly applied to nonholonomic systems [1,2]. In [3] a velocity observer is designed for output feedback tracking based on two-level tracking control architecture, i.e. on both kinematic and dynamic models derived in generalized coordinates.

To control nonholonomic systems a modeling framework other than the one utilizing generalized coordinates may be useful. This is due to differences between these systems from the control theory perspective and due to tasks they have to perform. Specifically, fully actuated nonholonomic systems may be controlled either at a kinematic or dynamic level and control variables are usually directly generalized velocities. Underactuated systems may be controlled at the dynamic level only and quasi-velocities used for modeling may easily specify control variables [4]. The two control groups are treated separately in the nonlinear control theory. In [5] it is demonstrated that one control-oriented modeling framework may be developed and nonholonomic systems modeling is unified with respect to it. The framework uses either generalized or quasi-coordinates to model nonholonomic systems. Also, a tracking control goal is usually meant as trajectory tracking. When other tasks become control goals, a control system modeling using quasi-coordinates may be more convenient than using generalized coordinates [4].

In the paper we present a model-based tracking control strategy extended by an incorporation of a nonlinear observer. The observer is designed to enable only position and orientation measurement for feedback tracking. The strategy is originally developed for programmed motion tracking when the full state of a system is available for measurement [5].

The contribution of the paper consists of designing a velocity observer for programmed motion tracking for a robot and its application to control dynamics developed either in generalized or quasi-coordinates. Some preliminary results on designing

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a velocity observer for a control dynamics developed in generalized coordinates are presented in [6]. Herein we employ quasi-coordinates to the robot dynamics description and exploit its kinematics of a unicycle. For this reason our approach cannot be directly applied to a general class of nonholonomic systems. However, kinematics of many mobile robots is equivalent to that of the unicycle and our result may be useful in applications.

The paper is organized as follows. Section 2 develops a dynamic control model for a mobile robot specified either in generalized or quasi-coordinates. Section 3 presents a nonlinear observer design. Section 4 develops a model-based controller–observer system that can track a desired motion of the robot. In Section 5 we detail an example of motion tracking for the mobile robot equipped with the velocity observer. The paper closes with conclusions and a list of references.

## 2. Control dynamics of a unicycle type mobile robot

A mobile robot model consists of a platform and two actuated wheels as presented in Fig. 1 [6]. Its geometric properties are specified in the figure.

A dynamic control model for the robot developed in generalized coordinates has the form

$$
\begin{aligned}
&M\ddot{q}_1 + C(\dot{q}_2)\dot{q}_1 + D\dot{q}_1 = \tau,\\
&\dot{q}_2 = B(q_2)\dot{q}_1.
\end{aligned}
\tag{1}
$$

The vector $q \in R^5$ is $q = (q_1, q_2)$, $q_1 \in R^2$, $q_2 \in R^3$, and $q_1 = (\varphi_r, \varphi_l)$ consists of driving wheel angles due to rolling, $q_2 = (x, y, \varphi)$, $x$, $y$ and $\varphi$ denote the position and orientation, respectively. The total mass consists of mass of the platform $m_c$ and of two wheels $2m_w$, i.e. $m = m_c + 2m_w$. $I_c$, $I_w$, $I_m$ are moments of inertia of the platform through the robot mass center, of the wheel about its axis and about its diameter, and $I = m_c d^2 + 2m_w b^2 + I_c + 2I_m$.

The inertia matrix is $M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$ and $m_{11} = \frac{r^2}{4b^2}(mb^2 + I) + I_w,\ m_{12} = \frac{r^2}{4b^2}(mb^2 - I),\ m_{21} = m_{12},\ m_{22} = m_{11}$.

The geometric parameters are: $r$ – wheel radius, $b$ – half of the platform width, $d$ – the distance along the axis $x$ between the robot mass center and the midpoint between the wheels.

The velocity dependent matrix is $C = \begin{bmatrix} 0 & c_{12} \\ -c_{12} & 0 \end{bmatrix}$ and its components are $c_{12} = \frac{r^2 d}{2b} m_c \dot{\varphi}$ and $c_{11} = c_{22} = 0$, $c_{12} = -c_{21}$.

The matrix of damping coefficients $D$ is $D - diag(d_{11}, d_{22})$. The control torque vector is $\tau \in R^2$, $\tau = (\tau_r, \tau_l)$. The $(3 \times 2)$ matrix $B$ of the nonholonomic material constraint equations is

$$
B = \begin{bmatrix} 0.5r\cos\varphi & 0.5r\cos\varphi \\ 0.5r\sin\varphi & 0.5r\sin\varphi \\ 0.5r/b & -0.5r/b \end{bmatrix}.
$$

The control dynamics (1) is derived with the control-oriented modeling framework developed in [5] so the equations are already in the reduced state form. It captures the specific structure of the robot model. The linear and angular velocities $v_r = \frac{r}{2}(\dot{\varphi}_r + \dot{\varphi}_l)$ and $\omega_r = \frac{r}{2b}(\dot{\varphi}_r - \dot{\varphi}_l)$ of the robot are not used in (1) as control variables. Instead, the angular velocities of wheels are the control variables in (1). Then, the dynamics in (1) has a convenient form to design a velocity observer.

For control applications it is more advantageous to develop the control dynamics (1) in quasi-coordinates using the Boltzmann–Hamel equations or their generalization [4]. To this end, we introduce a quasi-velocity vector $\Omega = (\omega, \omega^*)$, $\omega \in R^m$, $\omega^* \in R^k$, where $m = n - k$ is the number of degrees of freedom of a system and $k$ is the number of non-material, i.e. task-

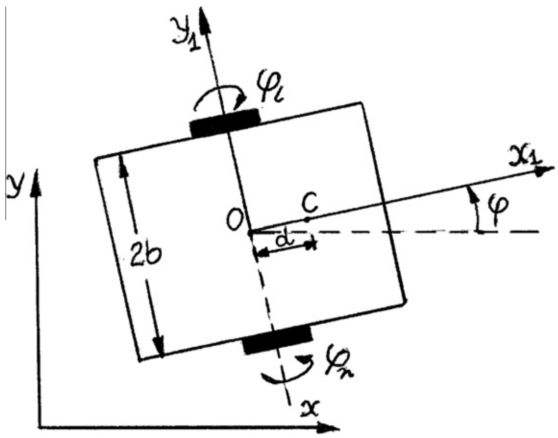


**Fig. 1.** Model of a two-wheeled mobile robot.