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Fixed-time coordinated tracking for second-order multi-agent systems with bounded input uncertainties

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ABSTRACT

In this paper, we study the fixed-time coordinated tracking problem for second-order integrator systems with bounded input uncertainties. Two novel distributed controllers are proposed with which the convergence time of the tracking errors is globally bounded for any initial condition of the agents. When relative state measurements are available for each follower, an observer-based distributed control strategy is proposed which achieves fixed-time coordinated tracking for the perturbed second-order multi-agent systems. When only relative output measurements are available, uniform robust exact differentiators are employed together with the observer-based controller which is able to achieve fixed-time coordinated tracking with reduced measurements. Simulation examples are provided to demonstrate the performance of the proposed controllers.

1. Introduction

Coordination control of multi-agent systems has been studied with great attention in recent years mainly due to its broad range of potential applications in areas such as coordination of multiple robots, unmanned aerial vehicles, autonomous underwater vehicles and spacecrafts [1,2]. Many control tasks have been considered such as consensus, leader-following, formation, swarming and flocking. Various types of distributed control laws have been proposed focusing on different agent dynamics and communication constraints. Readers are referred to the recent review articles [3–5] for more details.

In the distributed control of multi-agent systems, an important performance index of the control strategies is the convergence speed. Most of the existing control laws are asymptotical algorithms which means the coordination tasks can only be achieved as time approaches infinity. Motivated by the advantages of finite-time convergence laws such as faster convergence rate, higher precision and more robustness to uncertainties [6], finite-time coordination of multi-agent systems has attracted considerable attention recently. Based on the homogeneous controller design in [7], finite-time consensus control problem for first-order and second-order multi-agent systems are studied in [8,9], respectively. In [10], a continuous finite-time consensus controller was designed for double integrator systems using the adding a power integrator technique. Robust finite-time consensus tracking problem for multirobot systems was studied in [11] using nonsingular terminal sliding mode (NTSM) control method. Observe-based control strategies are proposed in [12–14] to achieve finite-time coordination for both low and high-order uncertain multi-agent system.

Note however, the convergence time of the above finite-time control laws dependent on the initial conditions of the agents. Therefore, a predefined convergence time cannot be guaranteed since the initial conditions of the agents is usually unavailable in advance. Motivated by this fact, several new results based on the notion of fixed-time stability [15] have appeared recently focusing on designing coordination control laws with guaranteed settling time regardless of the initial conditions of the agents. In [16] and [17], nonlinear fixed-time consensus algorithms are proposed for first-order integrator systems with undirected communication graphs. The results in [17] were further generalized in [18] to solve robust fixed-time consensus problems for first-order integrator systems with bounded input disturbances. In [19], the fixed-time cluster synchronization problem for complex networks is discussed with undirected communication topology. In [20], fixed-time leader-following problem was studied for first-order integrator systems with unknown nonlinear inherent dynamics under undirected communication graphs.

Due to the nonlinear nature of the fixed-time convergent controllers, it is very difficult to generalize the existing results for

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first-order integrator systems [16–20] to multi-agent systems with more complex agent dynamics. A first attempt is made in the recent paper [21] where the fixed-time consensus tracking problem for second-order multi-agent systems with directed communication graphs was studied based on terminal sliding mode control method. However, in [21], the control input of each follower depends directly on the inputs of its neighbors which leads to a loop problem when there exists cycles in the communication graph. Furthermore, uncertain dynamics and disturbances are not considered in the agents. In this work, we further consider the fixed-time coordinated tracking problem for second-order multi-agent systems with bounded input disturbances. Observer-based control strategies are proposed which achieve coordinated tracking with a guaranteed settling time independent on the initial conditions of the agents. The contribution of this work lies in the following aspects. First, compared with the existing results [16–20], distributed fixed-time coordinated control design is generalized to a larger class of multi-agent systems modeled as perturbed second-order systems. Second, the proposed controllers are truly distributed in the sense that only relative information is needed in the controller and no information about the global communication topology is required. Third, a novel fixed-time convergent NTSM controller for perturbed second-order integrator systems is developed as a basis for the distributed controller design. Finally, both the relative state measurements and relative output measurements cases are considered.

The rest of the paper is organized as follows. In Section 2, some preliminaries and the problem formulation are given. In Section 3, fixed-time coordinated tracking with relative state and output measurements are studied, respectively. In Section 4, coordinated tracking problem of multiple robotic manipulators is used as a simulation example to demonstrate the performance of the proposed controllers. Finally, some concluding remarks are given in Section 5.

Notation. $\mathbb{R}_+$ represents the set of positive real numbers. 0 is a vector or matrix with all the elements equal to zero. $\mathbf{1}_N \in \mathbb{R}^N$ is a vector with all the elements equal to 1. For a vector $x = (x_1, \ldots, x_n)$, $\|x\|_1$ and $\|x\|_\infty$ are the induced 1-norm and infinity-norm, respectively. $\mathrm{sgn}(\cdot)$ is the signum function and $\mathrm{sgn}(x) = (\mathrm{sgn}(x_1), \ldots, \mathrm{sgn}(x_n))$.

2. Preliminaries and problem setup

2.1. Graph theory

The communication relation among the agents in the leader–follower network can be represented by graphs. A directed graph $\mathcal{G} = (\mathcal{V}(\mathcal{G}), \mathcal{E}(\mathcal{G}))$ consists of a finite set of vertices $\mathcal{V}(\mathcal{G}) = \{e_0, e_1, \ldots, e_N\}$ and a finite set of edges $\mathcal{E}(\mathcal{G}) \subset \mathcal{V}(\mathcal{G}) \times \mathcal{V}(\mathcal{G})$. Each agent is represented by a vertex in $\mathcal{V}(\mathcal{G})$ and an edge is an ordered pair (e_i, e_j) which represents the information flow from agent j to agent i. The set of neighbors of e_i is denoted by $N_i = \{j : (e_i, e_j) \in \mathcal{E}(\mathcal{G})\}$. The degree of e_i is the number of its neighbors $|N_i|$ and is denoted by $\deg(e_i)$. A path $\mathcal{P}$ in $\mathcal{G}$ is a sequence $\{e_{i_0}, \ldots, e_{i_k}\}$ where $(e_{i_{j-1}}, e_{i_j}) \in \mathcal{E}(\mathcal{G})$ for $j = 1, \ldots, k$ and the vertices are distinct. Graph $\mathcal{G}$ is called undirected if (e_i, e_j) implies (e_j, e_i). An induced subgraph $\mathcal{G}_s$ of $\mathcal{G}$ is a graph such that $\mathcal{V}(\mathcal{G}_s) \subset \mathcal{V}(\mathcal{G})$ and for any $e_i, e_j \in \mathcal{V}(\mathcal{G}_s)$, $(e_i, e_j) \in \mathcal{E}(\mathcal{G}_s)$ if and only if $(e_i, e_j) \in \mathcal{E}(\mathcal{G})$. In this paper, we use the vertex set $\mathcal{V}(\mathcal{G}_s) = \{e_1, \ldots, e_N\}$ of subgraph $\mathcal{G}_s$ to represent the follower agents. The adjacency matrix $A = [a_{ij}]$ associated with $\mathcal{G}$ is defined as $a_{ii} = 0$ and $a_{ij} > 0$ if $(e_i, e_j) \in \mathcal{E}(\mathcal{G})$ where $i \neq j$. The Laplacian matrix of $\mathcal{G}$ is defined as $L = [l_{ij}]$ where $l_{ii} = \sum_{j \neq i} a_{ij}$ and $l_{ij} = -a_{ij}$ where $i \neq j$. Since the followers have no influence over the leader, we have $a_{0i} = 0, i = 1, \ldots, N$. Moreover $a_{i0}, i = 1, \ldots, N$ represents the communication relation between the leader and the followers where $a_{i0} > 0$ if follower i has information about the leader and $a_{i0} = 0$ otherwise. Let L_s denote the Laplacian matrix associated with $\mathcal{G}_s$, $B = \mathrm{diag}\{a_{10}, \ldots, a_{N0}\}$ and define

$$H = L_s + B, \tag{1}$$

then we have the following lemma about the property of H.

Lemma 1 (*[22]*). *If the subgraph $\mathcal{G}_s$ is undirected and each follower has a path to the leader in the graph $\mathcal{G}$, then H is symmetric and positive definite.*

2.2. Fixed-time stability

For the general differential equation

$$\dot{x} = f(t, x), \qquad x(0) = x_0 \tag{2}$$

where $x \in \mathbb{R}^n$ and $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function which may be discontinuous, the solutions of (2) are understood in the sense of Filippov [23]. Suppose the origin is an equilibrium point of (2).

Definition 1 (*[15]*). The origin of (2) is said to be globally finite-time stable if it is globally asymptotically stable and any solution $x(t, x_0)$ of (2) reaches the equilibria at some finite time moment, i.e., $x(t, x_0) = 0, \forall t \geq T(x_0)$, where $T : \mathbb{R}^n \to \mathbb{R}_+ \bigcup 0$ is the settling-time function.

The origin of the system $\dot{x} = -x^{1/3}$ is finite-time stable since any solution of the system converges to the origin in finite time $T(x_0) = (3/2)\sqrt[3]{|x_0|^2}$.

Definition 2 (*[15]*). The origin of (2) is said to be fixed-time stable if it is globally finite-time stable and the settling-time function $T(x_0)$ is bounded, i.e., $\exists T_{\max} > 0 : T(x_0) \leq T_{\max}, \forall x_0 \in \mathbb{R}^n$.

The origin of $\dot{x} = -x^{1/3} - x^3, x \in \mathbb{R}$ is fixed-time stable since it is globally finite-time stable and $x(t, x_0) = 0$ for $\forall t \geq 2.5$ and $\forall x_0 \in \mathbb{R}$.

2.3. Mathematical lemmas

Lemma 2 (*[18]*). *Let $\xi_1, \xi_2, \ldots, \xi_n \geq 0$ and $0 < p < 1$. Then*

$$\sum_{i=1}^{n} \xi_i^p \geq \left(\sum_{i=1}^{n} \xi_i\right)^p.$$

Lemma 3 (*[18]*). *Let $\xi_1, \xi_2, \ldots, \xi_n \geq 0$ and $p > 1$. Then*

$$\sum_{i=1}^{n} \xi_i^p \geq n^{1-p}\left(\sum_{i=1}^{n} \xi_i\right)^p.$$

Lemma 4 (*[17]*). *Consider a scalar system*

$$\dot{x} = -\alpha x^{2-p/q} - \beta x^{p/q}, \qquad x(0) = x_0, \tag{3}$$

where $\alpha, \beta > 0$, p, q are both positive odd integers satisfying $p < q$. Then, the equilibrium of Eq. (3) is finite-time stable and the settling time is bounded by

$$T(x_0) \leq \frac{q\pi}{2\sqrt{\alpha\beta}(q-p)}.$$

Lemma 5 (*[15]*). *If there exists a continuous radially unbounded function $V : \mathbb{R}^n \to \mathbb{R}_+ \bigcup 0$ such that (i) $V(x) = 0 \Rightarrow x = 0$; (ii) any*