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Topology selection for multi-agent systems with opposite leaders☆

Jingying Ma[a,b], Yuanshi Zheng[a,b], Long Wang[c,*]

[a] Key Laboratory of Electronic Equipment Structure Design of Ministry of Education, School of Mechano-electronic Engineering, Xidian University, Xi'an 710071, PR China

[b] Center for Complex Systems, School of Mechano-electronic Engineering, Xidian University, Xi'an 710071, PR China

[c] Center for Systems and Control, College of Engineering, Peking University, Beijing 100871, PR China

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ABSTRACT

For the multi-agent system with a navigational leader and its opponent, the followers cannot converge to the state of the navigational leader. In this paper, we consider the topology selection problem to minimize the opponent's influence which is measured by the tracking error of the system. Firstly, two combinatorial optimization problems are formulated. One is to minimize the tracking error by selecting guided informed-agents (the followers who can obtain the navigational leader's information). The other is to choose minimal number of guided informed-agents under an upper bound constraint of the tracking error. Secondly, for the scenario where the guided informed-agents are preset, we consider the problem of assigning the weights of edges to minimize the tracking error. Three convex optimization problems are proposed to evaluate the upper and lower bounds of the tracking error. Finally, numerical examples are provided to illustrate the effectiveness of the theoretical results.

1. Introduction

In the last decade, cooperative control of multi-agent systems (MASs) has captured tremendous attention from a wide range of academic disciplines, such as biology, physics, and social science etc [1–4]. Consensus seeking is an important issue of cooperative control of MASs which means all agents will converge to the same state. There have been extensive studies and results under various circumstances [5–10]. Multi-agent systems with leaders are also considered which leads to several research hotspots such as the leader-following consensus [11,12], containment control [13,14] and controllability analysis [15]. Shi et al. considered the leader-following consensus problem for MASs with a virtual leader [11]. In [12], the authors considered tracking control under variable topologies and obtained some sufficient conditions for solving the leader-following consensus. As an extension of consensus problem, containment control of multi-agent systems means that the followers will converge to the convex hull spanned by the leaders. Containment control under fixed undirected topology and switching topologies was considered in [13,16], respectively. Some sufficient and/or necessary conditions for solving containment control had also been addressed under varied models, such as containment control of heterogeneous MASs [17] and of MASs with measurement noises [18], etc.

In leader–follower multi-agent systems, followers can be classified into two types: the informed-agents who can receive information from leaders directly and the others who cannot. The leader sends its state information to the informed-agents who will spread this information to other followers by the interaction among the followers. Therefore, the interaction graph of the system is together determined by the subgraph of the followers and the set of informed-agents. Existing studies have shown that followers can converge to the leader under different interaction graphs. Then, a natural question that arises is how to design the interaction graph such that the system can converge quickly. The fast consensus problem is considered by solving semi-definite programming problems [5,19,20]. Based on linear-quadratic regulator theory, [21] proved that the optimal topology of leader-following consensus is a star topology. The problem of topology selection is also studied. In [22], the authors revealed that in order to achieve high accuracy, the more the agents, the smaller the proportion of informed-agents needed. [23] found that the convergence rate for first-order leader–follower MASs can also be determined by the

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* Corresponding author.
E-mail addresses: majy1980@126.com (J. Ma), zhengyuanshi2005@163.com (Y. Zheng), longwang@pku.edu.cn (L. Wang).

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maximal distance from the leader to the followers in the interaction graph. In order to minimize this distance, a standard combinatorial optimization problem was proposed. For leader–follower MASs with noises, [24] investigated the problem of minimizing the mean-square deviation via selecting a given number of informed-agents. Meanwhile, the problem of solving containment control in an optimal way was also studied. In [25], the authors gave an explicit expression to estimate the tracking error for multi-agent systems with some moving leaders. Clark et al. [26] formulated a leader selection problem for containment control in order to minimize the convergent error. A supermodular optimization approach was developed to solve this problem.

Competition and conflict are ubiquitous in practice. For the multi-agent system, differences of agents' interests may produce competition and opposition. One example from social science is election. Two candidates run for election and both want to beat their opponent. Based on this fact, [27] formulated the problem of maximizing influence in social networks with competitive ideas as a Stackelberg game. In [28], the authors provided a model to investigate the tension between information aggregation and spread of misinformation in large societies. Another example comes from networks. In a network, some nodes may be compromised by a malicious attacker whose objective is to disrupt the operation of the network. Therefore, a distributed strategy was developed to calculate any arbitrary function of the node values for networks with malicious nodes in [29]. In this paper, we consider the multi-agent system with two leaders who have opposite purposes. One leader named navigational leader propagates the navigational information to drive the followers to follow itself. The other leader named the opponent sends the misinformation to make the followers to keep away from the navigational leader. Considering the existence of the opponent, the followers will never converge to the navigational leader. Therefore, we focus on the problem of topology selection to reduce the opponent's influence. The main contribution of this paper is threefold. Firstly, we define the tracking error to quantify the opponent's influence. Then, we prove that if a follower is added into the set of guided informed-agents (who receive the navigational information), the tracking error will be decreased. Secondly, we formulate two topology selection problems. One is how to choose at most k guided informed-agents to minimize the tracking error. The other is how to select minimal number of guided informed-agents under an upper bound constraint of the tracking error. Since the two problems are *NP*-hard, two algorithms are developed to obtain their suboptimal solutions respectively. Finally, the problem of assigning the weights of edges is considered to reduce the opponent's influence for the case that the guided informed-agents are preset. We propose three convex programming problems to obtain the upper and the lower bounds of the minimal tracking error.

This paper is organized as follows. In Section 2, we introduce the graph theory and the system model. In Section 3, we formulate two guided informed-agents selection problems and develop two algorithms. The problem of designing the optimal weights of guided informed-edges to minimize tracking error is given in Section 4. In Section 5, numerical simulations are carried out to illustrate the effectiveness of our results. Some conclusions are drawn in Section 6.

Notation: Throughout this paper, the following notations will be used: let $\mathbb{R}$ be the set of real numbers. $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices. Denote $\mathbf{1}_n$ (or $\mathbf{0}_n$) as the column vector with all entries equal to one (or all zeros). For a column vector $\mathbf{b}=[b_1,b_2,\dots,b_n]^T$, $D_{\mathbf{b}}$ is a diagonal matrix with b_i, $i=1,\dots,n$, on its diagonal and $\|\mathbf{b}\|_p=(\sum_{i=1}^n|b_i|^p)^{\frac{1}{p}}$ is l^p-norm of $\mathbf{b}$. I_n denotes an n-dimensional identity matrix and $0_{n\times m}$ is a matrix with all zero entries. A matrix $A\in\mathbb{R}^{n\times m}$ is nonzero if $A\neq 0_{n\times m}$. For $A,B\in\mathbb{R}^{n\times m}$, denote $A>B$ (resp. $A\geq B$) if $A-B$ is a positive matrix (resp. nonnegative matrix), and let A_{ij} be the ij-th entry of A. For a square matrix A, ρ_A and $\mathrm{tr}(A)$ are the spectral radius and the trace of A, respectively. $\mathrm{adj}(A)$ is the adjugate of A. $\det(A)$ is the determinant of A. For a set S, $|S|$ and 2^S are the cardinality and the power set of S respectively. For two sets S_1 and S_2, denote $S_1\times S_2$ as the Cartesian product. $S_1\setminus S_2=S_1-S_2$. Let $\mathbf{e}_i$ denote the canonical vector with a 1 in the ith entry and 0's elsewhere.

2. Preliminaries

2.1. Graph theory

In this subsection, we present some basic notions of algebraic graph theory which will be used in this paper. For more details, interested readers are referred to [30] for a thorough study of graph theory.

Let $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{A}\}$ be an undirected graph consisting of a vertex set $\mathcal{V}=\{1,2,\dots,n\}$ and an edge set $\mathcal{E}=\{(i,j)\in\mathcal{V}\times\mathcal{V}\}$. The adjacency matrix $\mathcal{A}$ is a matrix such that for all $i\in\mathcal{I}_n$, $a_{ii}=0$ and for all $i\neq j$, $(i,j)\in\mathcal{E}$ if and only if $a_{ij}=a_{ji}>0$, while $a_{ji}=0$ otherwise. A walk of length r in a graph $\mathcal{G}$ is a sequence of vertices $i_0\sim i_1\sim\cdots\sim i_r$ where $(i_k,i_{k+1})\in\mathcal{E}$. If there exists at least one walk from the vertex i to j in $\mathcal{G}$ with length r, then $(\mathcal{A}^r)_{ij}>0$ [30]. The degree matrix $\mathcal{D}=D_{[d_1,\dots,d_n]^T}$ where $d_i=\sum_{j\in\mathcal{N}_i}a_{ij}$ and the Laplacian matrix $\mathcal{L}=\mathcal{D}-\mathcal{A}$.

Lemma 1. *Let $\mathcal{G}$ be a connected graph. Assume Δ be a nonnegative and nonzero diagonal matrix. Then, $\mathcal{L}+\Delta$ is positive definite and $(\mathcal{L}+\Delta)^{-1}$ is a positive matrix.*

Proof. By Lemma 3 in [12], we obtain that $\mathcal{L}+\Delta$ is positive definite. Let $\theta_i=(\mathcal{L}+\Delta)_{ii}$ and $\hat{\theta}=\max_{1\leq i\leq n}\theta_i$. It follows that

$$\mathcal{L}+\Delta=D_\theta-\mathcal{A}=\hat{\theta}\left[I_n-\left(\tilde{\Delta}+\frac{\mathcal{A}}{\hat{\theta}}\right)\right],$$

where $\theta=[\theta_1,\dots,\theta_n]$ and $\tilde{\Delta}=I_n-\frac{D_\theta}{\hat{\theta}}$. It is easy to show that $\tilde{\Delta}+\frac{\mathcal{A}}{\hat{\theta}}$ is a nonnegative matrix with spectral radius $\rho<1$. Hence, we have

$$(\mathcal{L}+\Delta)^{-1}=\frac{1}{\hat{\theta}}\sum_{k=0}^{\infty}\left(\tilde{\Delta}+\frac{\mathcal{A}}{\hat{\theta}}\right)^k\geq\frac{1}{\hat{\theta}}\sum_{k=0}^{\infty}\left(\frac{\mathcal{A}}{\hat{\theta}}\right)^k. \quad (1)$$

For every $(i,j)\in\mathcal{V}\times\mathcal{V}$, because $\mathcal{G}$ is connected, there exists at least one walk from i to j with length $k\geq 1$, i.e., $(\mathcal{A}^k)_{ij}>0$. Therefore, $\sum_{k=0}^{\infty}\mathcal{A}^k>0$. Together with (1), we have $(\mathcal{L}+\Delta)^{-1}>0$. ■

Lemma 2. *Let $\mathcal{G}$ be a connected graph. Let Δ_1,Δ_2 be two nonnegative and nonzero diagonal matrices. Suppose that $\Delta_2-\Delta_1\geq 0$ and $\Delta_2\neq\Delta_1$. Then, $(\mathcal{L}+\Delta_1)^{-1}-(\mathcal{L}+\Delta_2)^{-1}$ is a positive matrix.*

Proof. Denote $\Delta_2-\Delta_1=D_{[x_1,x_2,\dots,x_n]^T}$ and

$$L_0=\mathcal{L}+\Delta_1,$$
$$L_1=L_0+x_1\mathbf{e}_1\mathbf{e}_1^T,$$
$$\dots$$
$$L_n=L_{n-1}+x_n\mathbf{e}_n\mathbf{e}_n^T.$$

From the matrix inversion lemma [31], we have

$$L_{k-1}^{-1}-L_k^{-1}=\frac{L_{k-1}^{-1}x_k\mathbf{e}_k\mathbf{e}_k^TL_{k-1}^{-1}}{1+x_k\mathbf{e}_k^TL_{k-1}^{-1}\mathbf{e}_k},\quad k=1,2,\dots,n.$$