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Analytic approximate solutions for heat transfer of a micropolar fluid through a porous medium with radiation

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ABSTRACT

This paper aims to present complete analytic solution to heat transfer of a micropolar fluid through a porous medium with radiation. Homotopy analysis method (HAM) has been used to get accurate and complete analytic solution. The analytic solutions of the system of nonlinear ordinary differential equations are constructed in the series form. The convergence of the obtained series solutions is carefully analyzed. The velocity and temperature profiles are shown and the influence of coupling constant, permeability parameter and the radiation parameter on the heat transfer is discussed in detail. The validity of our solutions is verified by the numerical results (fourth-order Runge–Kutta method and shooting method).

1. Introduction

Modeling of natural phenomena and problems mostly leads to solving nonlinear equations. Solution of these equations is not easy especially through analytical approach. Therefore, endless of researchers are directed to decrease error in the solutions.

Due to mathematical complexity of Navier–Stokes equations, very few exact solutions can be obtained. In order to solve practical problems, different perturbation techniques have been widely used in engineering [1,2]. Mostly, these perturbation techniques lead to some important and attractive results; however they cannot be applied to all nonlinear problems. Perturbation techniques are based on the existence of small/large parameters, the so-called perturbation quantity. Unfortunately, many nonlinear problems in science and engineering do not contain such kind of perturbation quantities at all.

Some nonperturbative techniques, such as the artificial small parameter method [3], the δ-expansion method [4] and the Adomian's decomposition method [5], have been developed. Different from perturbation techniques, these nonperturbative methods are independent upon small parameters. However, both of the perturbation techniques and the nonperturbative methods cannot provide us with a simple way to adjust or control the convergence region and rate of given approximate series.

In 1992, Liao [6] employed the basic ideas of the homotopy in topology to propose a general analytic method for nonlinear problems, namely homotopy analysis method (HAM), [7–11]. Based on homotopy of topology, the validity of the HAM is independent of whether or not there exist small parameters in the considered equation. Therefore, the HAM can overcome the foregoing restrictions and limitations of perturbation methods [12]. Furthermore, the HAM always provides us with a family of solution expressions in the auxiliary parameter $\hbar$. The convergence region and rate of each solution might be determined conveniently by the auxiliary parameter $\hbar$. The HAM also avoids discretization and provides an efficient numerical solution with high accuracy, minimal calculation and avoidance of physically unrealistic assumptions. Besides, the HAM

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is rather general and contains the homotopy perturbation method (HPM) [11,12], the Adomian decomposition method (ADM) [13] and δ-expansion method. In fact, HPM and ADM are always special cases of HAM when $\hbar = -1$. The convergence of HAM solution series is dependent upon three factors, that is, the initial guess, the auxiliary linear operator, and the auxiliary parameter $\hbar$. However, as a special case of homotopy analysis method, the convergence of HPM solution series is only dependent upon two factors: the auxiliary linear operator and the initial guess. So, given the initial guess and the auxiliary linear operator, HPM cannot provide other ways to ensure that the solution is convergent. When the HPM is applied to a problem, a good enough initial approximation and/or auxiliary operator are necessary to ensure that the solution is convergent. However, as pointed out by Liao [7], there does not exist a general theory and an efficient approach to find a good enough initial guess for any a given nonlinear problem, especially when the nonlinear problems have multiple solutions, discontinuation, multi-value solutions, bifurcations and/or are unsteady.

In recent years, the homotopy analysis method has been successfully employed to solve many types of nonlinear problems such as the nonlinear equations arising in heat transfer [14], the nonlinear model of diffusion and reaction in porous catalysts [15], the chaotic dynamical systems [16], the non-homogeneous Blasius problem [17], the generalized three-dimensional MHD flow over a porous stretching sheet [18], the wire coating analysis using MHD Oldroyd 8-constant fluid [19], the axisymmetric flow and heat transfer of a second grade fluid past a stretching sheet [20], the MHD flow of a second grade fluid in a porous channel [21], the generalized Couette flow [22], the squeezing flow between two infinite plates [23], steady three-dimensional problem of condensation film on inclined rotating disk [24], the Burger and regularized long wave equations [25], the laminar viscous flow in a semi-porous channel in the presence of a uniform magnetic field [26], and other problems. All of these successful applications verified the validity, effectiveness and flexibility of the HAM.

The main goal of the present study is to find the totally analytic solution for heat transfer of a micropolar fluid through a porous medium with radiation by homotopy analysis method. This problem studied first by Abo-Eldahab and Ghonaim [27] in 2005 and exerted the similarity solution. In this way, the Letter has been organized as follows. In Section 2, the flow analysis and mathematical formulation are presented. In Section 3, we extend the application of the HAM to construct the approximate solutions for the governing equations. The convergence of the obtained series solutions is carefully analyzed in Section 4. Section 5 contains the results and discussion. The conclusions are summarized in Section 6.

2. Flow analysis and mathematical formulation

Fig. 1 shows the steady two-dimensional flow of an incompressible micropolar fluid through a porous medium past a continuously semi-infinite horizontal plat in the region $y > 0$. The governing equations for the boundary-layer to micropolar fluid through a porous medium are given as follow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + k_1\frac{\partial \sigma}{\partial y} + \frac{\upsilon\varphi}{K}(U - u) + C\varphi(U^2 - u^2), \tag{2.2}$$

$$G_1\frac{\partial^2 \sigma}{\partial y^2} - 2\sigma - \frac{\partial u}{\partial y} = 0, \tag{2.3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{2.4}$$

with boundary conditions

$$\begin{aligned} &u = 0, \quad v = 0, \quad \sigma = 0, \quad T = T_w, \quad \text{at } y = 0, \\ &u = U_0, \quad \sigma = 0, \quad T = T_\infty, \quad \text{at } y \to \infty, \end{aligned} \tag{2.5}$$

where $k_1 = \rho S$, $\upsilon = (\mu + S)/\rho$.

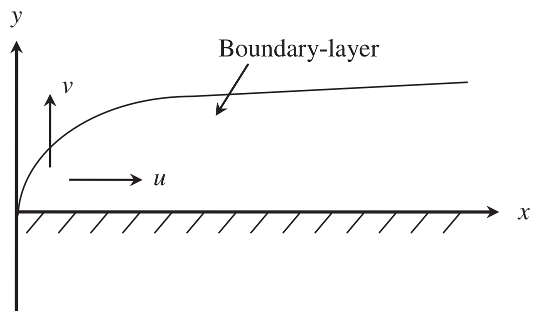

Fig. 1. Geometry of the problem.