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Adaptive output feedback consensus tracking for heterogeneous multi-agent systems with unknown dynamics under directed graphs

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ABSTRACT

This paper considers the distributed consensus tracking problem for the linear multi-agent systems with unknown dynamics under general directed graphs. Based on the output information among the agents, distributed adaptive consensus tracking protocols together with two observers, consisting of a local observer and an adaptive estimator, are designed to guarantee that all the signals in the closed-loop dynamics are uniformly ultimately bounded and the tracking errors converge to a small neighborhood around the origin. Moreover, the consensus protocols are in fully distributed fashion in the sense that the coupling gains in the controllers are independent without requiring the global knowledge that the eigenvalues of the Laplacian matrix associated with the whole communication graph. Finally, a simulation example is provided to verify the theoretical results.

1. Introduction

In recent years, coordination control of multi-agent systems has gained considerable interest partially due to its potential applications in many areas such as formation flying of unmanned air vehicles, cooperative surveillance, cooperative manipulation of multiple robots, distributed sensor networks [1–4], etc. In the previous literature, consensus control problem is a fundamental issue since the state agreement is often the premise for many coordination behaviors. The key objective of consensus control is to design distributed protocols, meaning that only the local state or output information is required, to achieve certain global task.

In the pioneering work [5], a framework of consensus control for the first-order integrators with different topologies and time delays has been established. Since then, the consensus control has been studied intensively in different directions such as quantized consensus [6,7], finite-time consensus [8,9], consensus with input or communication time delays [10,11], consensus with different dynamics [12–16]. In terms of the number of leaders, the above research works roughly fall into two classes, that is, leader-following consensus (or consensus tracking) with a reference to determine the final agreement value, and leaderless consensus whose agreement value depends on the initial states of the agents. Thus, the consensus tracking control has the advantages to determine the consensus value in advance, especially in some tasks, for example, to avoid hazardous obstacle.

In this paper, we consider the distributed consensus control problem for the general linear multi-agent systems with unknown dynamics in the agents. Related works on adaptive consensus control with unknown dynamics include [17–26], where the unknown nonlinear dynamics in the leader or the followers are linearly parameterized to design the consensus protocols. In [17], the authors considered the leaderless consensus problem for the single and double integrator-type systems with unknown dynamics under the undirected communication topology. Then in [18], the consensus tracking problem for the single integrator systems was solved with the directed graphs, but the controller required the relative velocity and position simultaneously. Under a strongly-connected communication graph, [19–21] studied the consensus tracking problem in a similar idea for the first-, second- and high-order integrator-type systems, respectively. In [22], adaptive consensus protocols were proposed for the single-integrator systems by exactly linearly parameterizing the unknown dynamics in the leader and followers. Among the aforementioned works, one common feature is that the consensus protocols are designed based on the state information which is not always available in practice. Moreover, another feature is that the dynamics of the agents focus on the first-order, second-order or high-order integrator-type systems which are special cases of the more general linear multi-agent systems. In [23,24], the output feedback

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consensus protocols were designed, respectively, for the double integrators and linear multi-agent systems with unknown dynamics. However, one limitation in [23,24] is that the control gains in the consensus protocols depend on the eigenvalues of the Laplacian matrix associated with the whole communication topology which is actually the global information of the network. Besides, such a limitation also exists in the controllers in [19–21]. By virtue of the adaptive coupling gain technique in [27–29], the state-feedback and output-feedback consensus tracking protocols have been designed in [25,26], respectively, without knowing the Laplacian matrices' eigenvalues. However, both of the protocols are only applicable to the undirected communication topology. Recently, adaptive consensus protocols have been designed under directed graphs for the second-order integrator-type systems [30] and general linear multi-agent systems [31], but both of them are state-feedback controllers. In [32], the authors proposed a new sequential observer approach to deal with the output feedback consensus for the linear multi-agent systems on directed graphs, but the dynamics focus on the nominal linear systems without considering the heterogeneous unknown dynamics in the followers.

Motivated by the aforementioned literature, in this paper, we aim to design fully distributed adaptive output-feedback consensus protocols for the multi-agent systems with unknown dynamics under the directed communication topology. Based on the output information among the agents, the observer-based adaptive protocols are designed for the leader-following consensus under the directed graph with a spanning tree. The main contributions of this paper are fourfold. Firstly, the protocols are designed for the general linear multi-agent systems which include the single, double, and high-order integrator-type systems [17–21] as its special cases. Secondly, compared with [17–21,25], in which the protocols are designed based on the state information among the agents, the observer-based adaptive controllers in this paper are developed by using the output information which is more practical. Moreover, the control gains in the protocols are independent of eigenvalues of the Laplacian matrix associated with the information-exchange graph, thus the protocols are in fully distributed fashion in the sense that only the local information is required without knowing global knowledge of the entire graph a priori. In [23,24], although the output-feedback protocols are developed, the parameters in the controllers are determined by the eigenvalues of the Laplacian matrix. Finally, the protocols in the present paper are applicable to more general directed graphs.

The paper is organized as follows. In Section 2, some preliminaries and the consensus problem are introduced. In Section 3, observer-based protocols are developed for the consensus tracking problem. A simulation example is given in Section 4 and the concluding remarks are provided in Section 5.

2. Preliminaries and problem statement

2.1. Notations

In this paper, let I_N and $\mathbf{1}$ denote the identity matrix of dimension N and a column vector with all entries equal to one, respectively. Let $R^{n\times m}$ represent a set of $n \times m$ real matrices, and $0_{n\times m}$ denote the matrices with all zeros. Given a real vector $x \in \mathbb{R}^n$, $\|x\|$ is the Euclidean norm of x, and for a matrix A, $\|A\|_F$ denotes the Frobenius norm that is defined by $\|A\|_F = \sqrt{\mathrm{tr}(A^TA)}$, where $\mathrm{tr}(\cdot)$ denotes the trace of a matrix. For a matrix P, $\lambda_{\min}(P)$, $\lambda_{max}(P)$ represent its minimum and maximum eigenvalue, respectively, and $\sigma_{\min}(P)$, $\sigma_{\max}(P)$ denote its minimum and maximum singular value, respectively. Given two matrices X and Y, $X \otimes Y$ denotes the Kronecker product of the matrices with the following properties that $\lambda(X \otimes Y) = \{\lambda_i(X)\lambda_j(Y)\}$ and $\sigma(X \otimes Y) = \{\sigma_i(X)\sigma_j(Y)\}$. $\mathrm{diag}(A_i)$ denotes a block-diagonal matrix with A_i, $i = 1, \ldots, N$, on the diagonal. Given two symmetric real matrices A and B, $A > B$ denotes that $A - B$ is positive definite.

2.2. Graph theory

In this paper, the communication topology among the agents is denoted by the directed graph $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} \triangleq \{1, \ldots, N\}$ is the node set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set consisting of ordered pair of distinct nodes and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacent matrix associated with the communication graph. In the directed graph $\mathcal{G}$, $(i, j) \in \mathcal{E}$ means that the jth agent has access to ith agent's information, but not vice versa. A directed path from node i_1 to node i_s is a sequence of edges of the form $(i_1, i_2), \ldots, (i_{s-1}, i_s)$. The communication graph $\mathcal{G}$ is strongly connected if there is a directed path from every node to the other node. A directed graph that contains a spanning tree is that there is a node called the root that has no parent node, and the root has a directed path to every other node of the graph. For the adjacent matrix $\mathcal{A} = [a_{ij}]$, $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$, otherwise is zero. It is assumed that there is no self-edge, that is, $a_{ii} = 0$ for all nodes. The Laplacian matrix $\mathcal{L} = [\mathcal{L}_{ij}]$ associated with the graph $\mathcal{G}$ is defined as $\mathcal{L}_{ii} = \sum_{j=0,j\neq i}^{N} a_{ij}$ and $\mathcal{L}_{ij} = -a_{ij}$, $i \neq j$.

Lemma 1 (*[33]*)**.** *The Laplacian matrix $\mathcal{L}$ associated with graph $\mathcal{G}$ has the property that it has a simple zero eigenvalue with vector* $\mathbf{1}$ *as a corresponding right eigenvector and all other eigenvalues have positive real parts if and only if $\mathcal{G}$ contains a directed spanning tree.*

Assumption 1. The graph $\mathcal{G}$ contains a directed spanning tree with the leader as the root node.

Because the leader has no neighbors, the Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ has the following structure

$$\mathcal{L} = \begin{bmatrix} 0 & 0_{1\times N} \\ \mathcal{L}_2 & \mathcal{L}_1 \end{bmatrix}, \tag{1}$$

where $\mathcal{L}_1 \in \mathbb{R}^{N\times N}$ and $\mathcal{L}_2 \in \mathbb{R}^{N\times 1}$. Under Assumption 1, zero is a simple eigenvalue of the Laplacian matrix $\mathcal{L}$ by virtue of Lemma 1. Thus it is obvious that $\mathcal{L}_1$ is a nonsingular M-matrix which has the following property:

Lemma 2 (*[31,34]*)**.** *For the nonsingular M-matrix $\mathcal{L}_1$, there exists a diagonal matrix $G = \mathrm{diag}(g_1, \ldots, g_N)$ with $g_i > 0$, $i = 1, \ldots, N$ such that $G\mathcal{L}_1 + \mathcal{L}_1^TG = \hat{\mathcal{L}}_1 > 0$. And the positive definite matrix G can be given with $[g_1, \ldots, g_N]^T = (\mathcal{L}_1)^{-1}\mathbf{1}$.*

Lemma 3 (*[27,35]*)**.** *For a system $\dot{x} = g(x, t)$, in which $g(\cdot)$ is locally Lipschitz in x and piecewise continuous in t, and it is assumed that there exists a continuously differentiable function $V(x, t) \geq 0$ such that the derivative along the trajectory of the system,*

$$\mathcal{K}_1(\|x\|) \leq V(x, t) \leq \mathcal{K}_2(\|x\|),$$
$$\dot{V}(x, t) \leq \mathcal{K}_3(\|x\|) + \epsilon$$

where the constant $\epsilon > 0$, $\mathcal{K}_1$, $\mathcal{K}_2$ belong to class $\mathcal{K}_\infty$ functions, and $\mathcal{K}_3$ belongs to class $\mathcal{K}$ function. Then, the solution $x(t)$ of the system is uniformly ultimately bounded.

Lemma 4. *For the matrices $W, \hat{W}, \tilde{W} \in \mathbb{R}^{m\times n}$ with the relation that $\tilde{W} = \hat{W} - W$, then*

$$\mathrm{tr}[\tilde{W}^T\hat{W}] = \frac{1}{2}\mathrm{tr}[\tilde{W}^T\tilde{W} + \hat{W}^T\hat{W} - W^TW].$$

Lemma 5 (*Young's Inequality [36]*)**.** *For nonnegative real numbers a, b, if p, q are real numbers that satisfy $\frac{1}{p} + \frac{1}{q} = 1$, then $ab \leq \frac{a^p}{p} + \frac{b^q}{q}$.*