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Frequency-domain subsystem identification with application to modeling human control behavior

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ABSTRACT

We present a frequency-domain subsystem identification algorithm that identifies unknown feedback and feedforward subsystems that are interconnected with a known subsystem. This method requires only accessible input and output measurements, applies to linear time-invariant subsystems, and uses a candidate-pool approach to ensure asymptotic stability of the identified closed-loop transfer function. We analyze the algorithm in the cases of noiseless and noisy data. The main analytic result of this paper shows that the coefficients of the identified feedback and feedforward transfer functions are arbitrarily close to the true coefficients if the data noise is sufficiently small and the candidate pool is sufficiently dense. This subsystem identification approach has application to modeling the control behavior of humans interacting with and receiving feedback from a dynamic system. We apply the algorithm to data from a human-in-the-loop experiment to model a human's control behavior.

1. Introduction

Humans learn to interact with many complex dynamic systems. For example, humans learn to drive cars, fly planes, and ride unicycles. Moreover, humans learn to control all of these systems with virtually no *a priori* information. The control strategies that humans learn and the approaches used to learn them are currently unknown. The internal model hypothesis proposes that humans construct models of their body and the physical world, and these models are used for control [1,2]. Although studies (e.g., [3–10]) provide evidence in support of the internal model hypothesis, this existing evidence is not conclusive [11].

Consider a scenario where a human interacts with a dynamic system by using feedback y_t and external information r_t (e.g., a command) to generate a control u_t as shown in Fig. 1. In this scenario, the human is an unknown subsystem, which can include both feedback and feedforward. Modeling the human's control strategy can be viewed as a subsystem identification (SSID) problem, where r_t and y_t are measured and the dynamic system with which the human interacts is assumed to be known. The internal signals that the human uses to construct u_t are inaccessible (i.e., unmeasurable). For example, if u_t is the sum of feedback and feedforward terms, then these individual terms are inaccessible.

Existing methods for SSID are given in [12–24]. Specifically, [12–14] present methods for static subsystems, while [15–24] present methods for dynamic subsystems. In the dynamic SSID literature, the approaches in [15–19] are restricted to open-loop SSID, that is, identification of subsystems interconnected without feedback. We note that [16,19] use open-loop SSID to model the dynamics of human subsystems. Specifically, [16] identifies a transfer function that models a human's precision grip force dynamics, whereas [19] identifies two transfer functions that together model a human's oculomotor subsystem.

In contrast to [12–19], we focus on dynamic closed-loop SSID, that is, identification of dynamic subsystems with feedback. Existing dynamic closed-loop SSID methods include [20–24]. In particular, [20] identifies a transfer function that models the behavior of a human subject interacting in feedback with a mechanical system. However, the method in [20] applies to systems with feedback only, that is, systems without feedforward. We note that the methods in [20–24] are time-domain techniques and yield identified models that may not result in an asymptotically stable closed-loop system.

This paper presents a new closed-loop SSID technique that: (i) identifies feedback and feedforward subsystems, and (ii) ensures asymptotic stability of the identified closed-loop transfer function. A closed-loop SSID method that addresses both (i) and (ii) is a new contribution of this paper. The method relies on a candidate-pool approach to accomplish (ii). Another contribution of this paper is an analysis of the properties of the SSID algorithm in the cases

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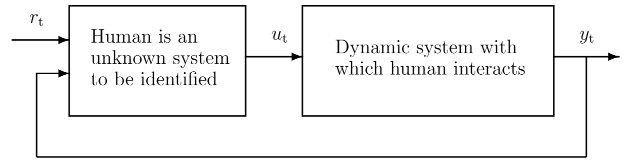

Fig. 1. Modeling a human's control strategy can be viewed as an SSID problem, where r_t and y_t are measured and the dynamic system with which the human interacts is assumed to be known.

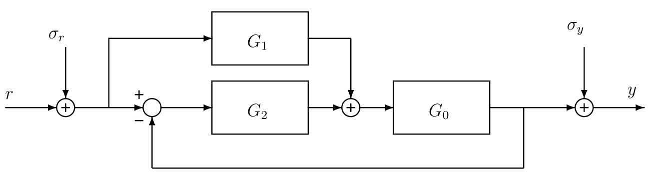

Fig. 2. The input r and output y of this linear time-invariant system are measured, but all internal signals are inaccessible.

of noiseless and noisy data. Our main analytic result shows that the coefficients of the identified feedback and feedforward transfer functions are arbitrarily close to the true coefficients if the data noise is sufficiently small and the candidate pool is sufficiently dense.

Characteristics (i) and (ii) of the SSID algorithm are motivated by the application to modeling human control behavior. First, humans generally use both anticipatory (feedforward) and reactive (feedback) control [1,2], which motivates (i). Second, if a human-in-the-loop system has a bounded output, then it is desirable to identify subsystems that result in an asymptotically stable closed-loop transfer function, thus motivating (ii). In addition, human control behavior is band limited; specifically, humans cannot produce motion with arbitrarily high frequency. Thus, we are interested in identifying models over a limited frequency range, which motivates the development of a new SSID technique in the frequency domain.

The frequency-domain SSID technique in this paper identifies unknown feedback and feedforward subsystems that are interconnected with a known subsystem, where all internal signals are assumed to be inaccessible, as shown in Fig. 2. We present numerical examples to demonstrate the properties of the SSID algorithm. We also apply the SSID algorithm to data from an experiment designed to model the control behavior of a human subject interacting with a dynamic system. System identification approaches are also used in [25–31] to model human control behavior. However, [25–31] do not model both feedback and feedforward subsystems and do not ensure stability of the identified closed-loop model. A preliminary version of the SSID algorithm in this paper appeared in the conference proceedings [32]. However, this paper goes beyond the work of [32] by analyzing the properties of the SSID algorithm.

2. Problem formulation

Consider the linear time-invariant system shown in Fig. 2, where r, y, σ_r, and σ_y are the Laplace transforms of the input, output, input noise, and output noise, respectively, and for $i = 0, 1, 2$, $G_i : \mathbb{C} \to \mathbb{C}$ is a real rational transfer function. If $\sigma_r = 0$ and $\sigma_y = 0$, then the closed-loop transfer function from r to y is given by

$$\tilde{G}(s) \triangleq \frac{G_0(s)G_1(s) + G_0(s)G_2(s)}{1 + G_0(s)G_2(s)}. \tag{1}$$

Next, let N be a positive integer, and define $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$. For all $k \in \mathcal{N}$, let $\omega_k \in (0, \infty)$, where $\omega_1 < \cdots < \omega_N$. Furthermore, for all $k \in \mathcal{N}$, define the *closed-loop frequency response data*

$$H(\omega_k) \triangleq \frac{y(\jmath\omega_k)}{r(\jmath\omega_k)} = \tilde{G}(\jmath\omega_k) + \sigma(\jmath\omega_k), \tag{2}$$

where $\sigma(s) \triangleq [\tilde{G}(s)\sigma_r(s) + \sigma_y(s)]/r(s)$. If $\sigma(\jmath\omega_k) \equiv 0$, then $\{H(\omega_k)\}_{k=1}^N$ is noiseless. In contrast, if $\sigma(\jmath\omega_k) \not\equiv 0$, then $\{H(\omega_k)\}_{k=1}^N$ is noisy.

We present a method to identify G_1 and G_2 provided that G_0 and $\{H(\omega_k)\}_{k=1}^N$ are known and $G_0 \neq 0$. In this case, the closed-loop frequency response data $\{H(\omega_k)\}_{k=1}^N$ can be obtained from the accessible signals r and y and does not depend on the internal signals, which are not assumed to be measured.

For $i = 0, 1, 2$, G_i can be expressed as $G_i(s) = N_i(s)/D_i(s)$, where N_i and D_i are coprime, and D_i is monic. The degrees of N_i and D_i are denoted by $n_i \triangleq \deg N_i$ and $d_i \triangleq \deg D_i$. Thus, (1) can be expressed as

$$\tilde{G}(s) = \frac{N_0(s)\,[D_2(s)N_1(s) + D_1(s)N_2(s)]}{D_1(s)\,[D_0(s)D_2(s) + N_0(s)N_2(s)]}.$$

We make the following assumptions:

(A1) d_1, d_2, n_1, and n_2 are known.
(A2) $d_0 + d_2 > n_0 + n_2$.
(A3) $N > d_0 + d_1 + d_2 + n_0 + \max\{n_1 + d_2, n_2 + d_1\}$.
(A4) If $\lambda \in \mathbb{C}$ and $D_1(\lambda)\,[D_0(\lambda)D_2(\lambda) + N_0(\lambda)N_2(\lambda)] = 0$, then $\mathrm{Re}\lambda < 0$.

Assumption (A1) can be replaced by the assumption that upper bounds on d_1, d_2, n_1, and n_2 are known. However, we invoke (A1) for clarity of the presentation. Assumption (A2) states that the loop transfer function G_0G_2 is strictly proper. Assumption (A3) implies that the number N of frequency response data points is sufficiently large. This assumption ensures that the minimization problem solved in the SSID has a unique solution. Assumption (A4) implies that $\tilde{G}$ is asymptotically stable, that is, the poles of $\tilde{G}$ are in the open-left-half complex plane.

Define $d \triangleq d_1 + d_2 + n_2 + 1$, and for all nonnegative integers j, let $\Gamma_j : \mathbb{C} \to \mathbb{C}^{j+1}$ be given by $\Gamma_j(s) \triangleq [s^j \quad s^{j-1} \quad \cdots \quad s \quad 1]^{\mathrm{T}}$. Consider the functions $\mathcal{N}_1 : \mathbb{C} \times \mathbb{R}^{n_1+1} \to \mathbb{C}$ and $\mathcal{D}_1, \mathcal{N}_2, \mathcal{D}_2 : \mathbb{C} \times \mathbb{R}^d \to \mathbb{C}$ given by

$$\mathcal{N}_1(s, \beta) \triangleq \Gamma_{n_1}^{\mathrm{T}}(s)\beta, \qquad \mathcal{D}_1(s, \phi) \triangleq s^{d_1} + \Gamma_{d_1-1}^{\mathrm{T}}(s)E_1\phi,$$
$$\mathcal{N}_2(s, \phi) \triangleq \Gamma_{n_2}^{\mathrm{T}}(s)E_2\phi, \qquad \mathcal{D}_2(s, \phi) \triangleq s^{d_2} + \Gamma_{d_2-1}^{\mathrm{T}}(s)E_3\phi,$$

where $E_1 \triangleq [I_{d_1} \quad 0_{d_1\times(d_2+n_2+1)}] \in \mathbb{R}^{d_1\times d}$, $E_2 \triangleq [0_{(n_2+1)\times d_1} \quad I_{n_2+1} \quad 0_{(n_2+1)\times d_2}] \in \mathbb{R}^{(n_2+1)\times d}$, $E_3 \triangleq [0_{d_2\times(d_1+n_2+1)} \quad I_{d_2}] \in \mathbb{R}^{d_2\times d}$, $\beta \in \mathbb{R}^{n_1+1}$, and $\phi \in \mathbb{R}^d$. Next, consider the functions $\mathcal{G}_1 : \mathbb{C} \times \mathbb{R}^{n_1+1} \times \mathbb{R}^d \to \mathbb{C}$ and $\mathcal{G}_2 : \mathbb{C} \times \mathbb{R}^d \to \mathbb{C}$ given by

$$\mathcal{G}_1(s, \beta, \phi) \triangleq \frac{\mathcal{N}_1(s, \beta)}{\mathcal{D}_1(s, \phi)}, \qquad \mathcal{G}_2(s, \phi) \triangleq \frac{\mathcal{N}_2(s, \phi)}{\mathcal{D}_2(s, \phi)},$$

which, for each $\beta \in \mathbb{R}^{n_1+1}$ and $\phi \in \mathbb{R}^d$, are real rational transfer functions.

Our objective is to determine β and ϕ such that $\mathcal{G}_1$ and $\mathcal{G}_2$ approximate G_1 and G_2, respectively. To achieve this objective, consider the cost function

$$J(\beta, \phi) \triangleq \sum_{k=1}^{N} \left| \frac{N_0(\jmath\omega_k)\,[\mathcal{D}_2(\jmath\omega_k, \phi)\mathcal{N}_1(\jmath\omega_k, \beta) + \mathcal{D}_1(\jmath\omega_k, \phi)\mathcal{N}_2(\jmath\omega_k, \phi)]}{\mathcal{D}_1(\jmath\omega_k, \phi)\,[D_0(\jmath\omega_k)\mathcal{D}_2(\jmath\omega_k, \phi) + N_0(\jmath\omega_k)\mathcal{N}_2(\jmath\omega_k, \phi)]} - H(\omega_k) \right|^2, \tag{3}$$

which is a measure of the difference between the closed-loop frequency response data $\{H(\omega_k)\}_{k=1}^N$ and the closed-loop transfer