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Dissipativity and feedback passivation for switched discrete-time nonlinear systems

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ABSTRACT

This paper studies dissipativity for switched discrete-time nonlinear systems using multiple storage functions and multiple supply rates. A sufficient condition for dissipativity of the switched nonlinear system is given under some switching law. Then, the result is extended to find a condition under which a switched system is feedback equivalent to a passive switched system. Switched passivity condition and switched l_2-gain inequalities are, respectively, given, which are generalizations of the classical ones. Furthermore, the assumption of zero dynamics having passivity is relaxed. Passivity is also shown to be preserved under feedback interconnection.

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1. Introduction

Dissipativity as one of the most important and desirable system properties, is a very effective tool for the study of nonlinear systems. In many engineering problems, stability, tracking issues and control synthesis are often linked to dissipative systems. A dissipative system is a system for which the energy dissipated inside the system is no more than the supplied energy from the external source. The dissipativity is characterized by storage functions and supply rates.

The dissipativity concept and dissipativity theory were developed by Willems [1]. The extensions of these results to the case of affine nonlinear systems were carried out in [2–4]. Since dissipative (passive) systems present highly desirable properties, the dissipativity (passivity) has become one of the major approaches to the study of complex systems, which may simplify systems analysis and control design [2]. For continuous-time nonlinear systems, the concepts of dissipativity and passivity have been widely used to solve stability and stabilization problems [5–7], and control synthesis with successful applications in many systems including electronic-type and electromechanical systems (see, for example [8,9]). Results on dissipativity and passivity of discrete-time nonlinear systems have also appeared. In [10–12], Kalman–Yakubovich–Popov (KYP) conditions were extended to discrete-time nonlinear systems. Feedback losslessness equivalence and feedback passivity equivalence for discrete-time nonlinear systems were investigated in [13,14], respectively.

On the other hand, since switched systems are one important and particular class of hybrid systems, the study of switched systems has received much attention (see, for example [15–18]). The main methodologies used in studying switched systems are multiple Lyapunov functions [19,20], (average) dwell time [21–24] and so on. Recently, some less conservative methods have been proposed, such as general multiple Lyapunov functions method [25], general (average) dwell time technique [26], and dwell time min-switching approach [27,28] and so on.

As mentioned earlier, dissipativity (passivity) is an important property and a powerful tool for non-switched nonlinear systems. Naturally, the dissipativity (passivity) property for hybrid and switched systems is still expected to be useful. There are some papers concerning dissipativity and passivity-based control problem of switched systems and hybrid systems [29–36]. For switched continuous-time systems, [32] investigated passivity and passivity-based controller design via a common storage function. [37] proposed a notion of passivity by using multiple storage functions. However, this passivity concept requires each storage

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function to be non-increasing on the consecutive "switched on" times. [38] given a framework of dissipativity theory for switched continuous-time systems using multiple storage functions and multiple supply rates, and considered the changes of energy not only for the activated but also for inactivated subsystems.

However, dissipativity and passivity of switched discrete-time systems have been rarely explored. The existing works only consider switched systems with all modes or at least one mode being dissipative (passive). The use of multiple storage functions for different modes of a switched linear system was proposed via piecewise quadratic storage functions [39]. Recently, a new concept of decomposable dissipativity was proposed in [40]. By assuming the dissipativity of each subsystem, only the stability conditions were given in [40]. However, neither conditions to guarantee the dissipativity nor control law design to achieve dissipativity were considered. This motivates the study of this paper. On the other hand, [41] extended the results of [40] to the case where not all subsystems are passive, but at least one is passive, while no exchange of energy between the active subsystem and those inactivated subsystems was considered. However, the proposed approach used in the above results cannot handle the case where all subsystems are non-dissipative (non-passive). Another motivation of our present study comes from [38]. This paper can be seen as a parallel result of [38] to the discrete-time counterpart. However, the extension is nontrivial due to some distinctive features of discrete-time switched systems.

In this paper, we are interested in dissipativity (passivity) of switched discrete-time nonlinear systems in the absence of dissipativity of subsystems. On one hand, for a switched discrete-time system which is nonlinear non-affine in the input, by designing a proper switching rule, a sufficient condition for the system to be dissipative is given. On the other hand, the problem of rendering a switched system to be passive by means of a static feedback control law under some switching law is presented, and a condition for passification of a switched discrete-time nonlinear system is obtained without the requirement of passivity of its zero dynamics. Moreover, for passivity, we localize the passivity condition and obtain a property of invariance under feedback interconnection. More importantly, the switching signals of the two switched systems of the interconnected switched system can be different. Therefore, the two switched systems are allowed to be switched asynchronously. This provides more freedom for design. For l_2-gain, we derive a local version of the Hamilton–Jacobi Inequalities (HJIs), which is a generalization of the classical one.

2. Preliminaries

Consider the switched discrete-time nonlinear system

$$\begin{cases} x(k+1) = f_{\sigma(k)}(x(k), u_{\sigma(k)}(k)) \\ y(k) = h_{\sigma(k)}(x(k), u_{\sigma(k)}(k)), \end{cases} \tag{1}$$

where $k \in N$, $\sigma(k)$ is the switching signal taking values in $I = \{1, 2, \dots, M\}$; $x(k) \in X \subset R^n$ is the state vector, $u_i(k) \in U_i \subset R^{m_i}$ is the input vector of the ith subsystem, and $y(k) \in Y \subset R^m$ is the output vector. X, U_i and Y are the state, input, and output spaces, respectively. Both $f_i : R^n \times R^{m_i} \to R^n$ and $h_i : R^n \times R^{m_i} \to R^m$ are C^2 functions. All considerations are restricted to an open set of $X \times U_i$ containing the equilibrium point (x^*, u_i^*). We assume $f_i(0, 0) = 0$ and $h_i(0, 0) = 0$.
For the system (1), we make the following assumption.

Assumption 1. $\left.\frac{\partial h_i(x,u_i)}{\partial u_i}\right|_{(x^*,\, u_i^*)} \neq 0$, that is, each subsystem has local relative degree zero.

Remark 1. Assumption 1 is a common prerequisite for dissipativity study of discrete-time systems [10,13].

In this paper, we focus on the following main problems.

(a) When all subsystems are non-dissipative, how to achieve dissipativity for the system (1) via design of a proper switching rule.
(b) When the zero dynamics of all subsystems of the system (1) are non-passive, how to achieve feedback passivation for a nonlinear system affine in the control input via design of subsystem controllers and a certain switching rule.

3. Dissipativity

This section will study the dissipativity for the system (1) by using multiple storage functions and multiple supply rates. We will provide a sufficient condition for the system (1) to be dissipative under some switching rule. First, inspired by the concept of dissipativity for switched continuous-time systems [38], we will present the following dissipativity definition for switched discrete-time systems.

Definition 1. System (1) is said to be locally dissipative under some switching signal $\sigma(k)$ if there exist positive semidefinite continuous functions $V_i(x)$, $i \in I$, called storage functions, locally completely summable functions $s_i(y, u_i)$, $i \in I$, called supply rates, and locally completely summable functions $\phi_r^i(x, y, u_i, k)$, r, $i \in Ir \neq i$, called cross-supply rates, such that the following inequalities hold

$$\begin{aligned} \Delta V_i(x(k)) &= V_i(f_i(x(k),\ u_i(k))) - V_i(x(k)) \\ &\le s_i(y(k), u_i(k)), \quad \sigma(k) = i,\ \forall x \in X,\ u_i \in U_i \end{aligned} \tag{2}$$

$$\begin{aligned} \Delta V_r(x(k)) &= V_r(f_i(x(k), u_i(k))) - V_r(x(k)) \\ &\le \phi_r^i(x(k), y(k), u_i(k), k), \\ &\quad r \neq i,\ \forall x \in X,\ u_i \in U_i. \end{aligned} \tag{3}$$

Similar to the discussion of [38], since all subsystems share the same state variable, the energy $V_r(x)$ of the inactive rth subsystem is actually changing. This can be regarded as the result of imported energy from the active ith subsystem into the inactive rth subsystem. This energy is described by the cross-supply rate ϕ_r^i from the ith subsystem to the rth subsystem and fulfils the dissipation inequalities (3).

Remark 2. Definition 1 can be as a discrete version of the dissipativity concept of [38].

We now give a sufficient condition for the switched system (1) to be dissipative. Before doing so, we make the following assumption for the storage functions V_i, the supply rates $s_i(y, u_i)$ and the cross-supply rates $\phi_r^i(x, y, u_i, k)$ which will be used in the following results.

Assumption 2. $V_i(f_i(x, u_i))$, $s_i(y, u_i)$ and $\phi_r^i(x, y, u_i, k)$ are all C^2 and quadratic with respect to control.

Theorem 1. *Let $V_i(x)$, $s_i(y, u_i)$ and $\phi_r^i(x, y, u_i, k)$ be storage functions, supply rates and cross supply rates, respectively, all being quadratic in u_i. If the following conditions hold*

$$V_i(f_i(x, 0)) - V_i(x) + \sum_{r=1, r\neq i}^{M} \beta_{ir}(V_r(x) - V_i(x)) \le s_i(h_i(x, 0), 0), \tag{4a}$$

$$\left.\frac{\partial V_i(a)}{\partial a}\right|_{a=f_i(x,0)} \left.\frac{\partial f_i(x, u_i)}{\partial u_i}\right|_{u_i=0} = \left.\frac{\partial s_i(h_i(x, u_i),\ u_i)}{\partial u_i}\right|_{u_i=0} + \left.\frac{\partial s_i(h_i(x, u_i),\ u_i)}{\partial y} \frac{\partial h_i(x,\ u_i)}{\partial u_i}\right|_{u_i=0}, \tag{4b}$$

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