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Periodic event-triggering in distributed receding horizon control of nonlinear systems

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ABSTRACT

How to efficiently use limited system resources in distributed receding horizon control (DRHC) is an important issue. This paper studies the DRHC problem for a class of dynamically decoupled nonlinear systems under the framework of event-triggering, to efficiently make use of the computation and communication resources. To that end, a distributed periodic event-triggered strategy is designed and a detailed DRHC algorithm is presented. The conditions for ensuring feasibility of the designed algorithm and stability of the closed-loop system are developed, respectively. We show that the closed-loop system is input-to-state stable if the energy bound of the disturbances, the triggering condition and the cooperation matrices fulfill the proposed conditions.

1. Introduction

The distributed receding horizon control (DRHC) finds many applications in multi-agent systems, smart-grids, energy systems, chemical processes, and so on. In the past few years, many results on DRHC are developed with many different focuses. Generally speaking, the results can be categorized into two classes according to the characteristics of the large-scale systems, namely, the large-scale systems with coupled subsystems (see, e.g., [1,2]) and those with decoupled subsystems (see, e.g., [3,4]). The second type of study can be used for cooperative control, formation control, and synchronization of multi-agent systems such as multiple robots, aircraft fleet, multiple sensor systems, which is of great practical interest and is the focus of current study.

Although great progresses have been made for solving the DRHC problem, an important practical issue still remains unaddressed: How to use limited computation and network resources in a large-scale system to achieve acceptable control performance? So far, most of the results of DRHC use fixed periods to update control input signal and exchange information, which is NOT computational efficient according to the event-triggered control strategy [5]. Thus, in this paper, we will study the DRHC problem under the framework of Lebesgue sampling (event based sampling) rather than Riemann sampling (conventional sampling with fixed period), aiming at achieving desired control performance using limited system resources.

In 2002, an important result on event-triggered control was developed in [5]. Since then, there have been increasing interest for studying event-triggered control, for example, [6,7]. Of particular interest, there have been several results on receding horizon control (RHC) using event-triggered strategy, which are reviewed as follows. In [8], an event-triggered control strategy for discrete-time linear system subject to disturbance is presented. In [9], Grüne et al. propose to use the linearization error to trigger the events, determining time instants when an optimization problem needs to be solved. For linear systems, a self-triggered RHC strategy is proposed in [10] to avoid continuous testing on triggering condition. In [11], an event-triggered RHC scheme is established for discrete-time nonlinear systems, where the triggering condition is designed to guarantee the input-to-state stability (ISS), and the result in [11] is further extended for decentralized RHC in [12]. The event-based distributed RHC for agent cooperation is studied in [13], extended to [14] by using self-triggered strategy. The event-triggered RHC stabilization problem of nonlinear systems without disturbances and with disturbances is reported in [15] and [16], respectively.

Recently, a novel strategy called periodic event-triggered strategy is proposed in [17], which requires only periodically testing the triggering conditions, and thus is more practical and efficient. The periodic event-triggered control problem of

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nonlinear systems is investigated in [18], where the nonlinear dynamics is assumed to be p-times continuous differentiable, and the pth Lie derivative of the triggering-condition function along the system dynamics needs to be upper-bounded by a function. In comparison with [18], the nonlinear dynamics in this study is assumed to be twice continuously differentiable, and a state feedback control law needs to exist for the linearized system. In this paper, we consider the DRHC problem of a class of dynamically decoupled *nonlinear systems* under the framework of *periodic event-triggering*, aiming at developing a practical and efficient nonlinear DHRC approach. This paper extends the result in [19] to nonlinear system case, capturing it as a special case. The main contributions of this study are two-fold:

- A periodic event-triggered DRHC scheme is proposed for a class of large-scale nonlinear systems. In the new DRHC scheme, we formulate a new optimization problem for each agent and then design a detailed periodic triggering strategy for each agent to determine the time instant when the optimal control input should be generated. The designed scheme reduces the communication and computation load by taking advantage of the event-triggered control, and only requires periodically testing the triggering conditions, which is practically useful.
- The theoretical analysis of the designed DRHC scheme is conducted. We establish sufficient conditions for ensuring the feasibility of the designed DRHC algorithm for each agent. In addition, we develop sufficient conditions under which the overall system is stable. It is shown that the closed-loop system is input-to-state stable, if the energy bound of the disturbances, the triggering level and the cooperation matrices satisfy the proposed conditions.

Notations: $\mathbb{N}$ represents the set of positive integers, and $\mathbb{R}^n$ denotes the n-dimensional real space. For a matrix P, $P > 0$ ($P \geqslant 0$) means the matrix is positive definite (positive semi-definite), and P^{T} and P^{-1} stands for its transpose and inverse, respectively. $\forall x \in \mathbb{R}$, $\lceil x \rceil$ means the smallest integer which is great than or equal to x. A diagonal matrix P with elements $x_1, x_2, \dots, x_n$ is denoted as $P = \operatorname{diag}(x_1, x_2, \dots, x_n)$. For a vector $v \in \mathbb{R}^n$, denote its 2-norm by $\|v\|$, and its P-weighted norm as $\|v\|_P \triangleq \sqrt{v^{\mathrm{T}} P v}$, with P being a given matrix with appropriate dimension. Given a matrix Q, the maximum and minimum of the absolute values of its eigenvalues are denoted by $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$, respectively. Given two matrices Q and P, $\lambda_{Q,P} \triangleq \frac{\lambda_{\max}(Q)}{\lambda_{\min}(P)}$.

2. Problem formulation

Consider a multi-agent nonlinear system of M agents. Each agent i is described as

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t)) + \omega_i(t), \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the system state, $u_i(t) \in \mathbb{R}^m$ is the control input, $\omega_i(t) \in \mathbb{R}^n$ is the disturbance. Due to the actuator saturation, $u_i(t) \in \mathcal{U}_i$, where $\mathcal{U}_i \subseteq \mathbb{R}^m$ is a compact set and contains zero as its interior point. $\omega_i(t) \in \mathcal{W}_i$ with $\mathcal{W}_i$ being a compact set, and define $\rho_i \triangleq \sup_{\omega_i(t) \in \mathcal{W}_i} \|\omega_i(t)\|$.

There is a communication network connecting this multi-agent system, through which each agent can exchange information with some agents in its neighboring area. For agent i, define its neighbors as the agents from which it can receive information. The set of indices of agent i's neighbors is denoted by $\mathcal{N}_i$. Assume that each agent has at least one neighbor. To facilitate the DRHC design, two standing assumptions are made [20,21].

Assumption 1. For each agent i, suppose (A.1) $f_i : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is twice continuously differentiable, and $f_i(0, 0) = 0$; (A.2) the control input $u_i(t) \in \mathcal{U}_i$ is piecewise right-continuous; (A.3) for any initial value $x_i(0) \in \mathbb{R}^n$, piecewise right-continuous $u_i(t) \in \mathcal{U}_i$ and disturbance $\omega_i(t) \in \mathcal{W}_i$, $t \geqslant 0$, the differential equation in (1) admits a unique solution.

The nominal system of the system in (1) is characterized as $\dot{\bar{x}}_i(t) = f_i(\bar{x}_i(t), u_i(t))$. By linearizing the nominal system at (0, 0), we can get $\dot{\bar{x}}_i(t) = A_i \bar{x}_i(t) + B_i u_i(t)$, where $A_i = \partial f_i / \partial x_i|_{(0,0)}$ and $B_i = \partial f_i / \partial u_i|_{(0,0)}$.

Assumption 2. There exists a control law $u_i(t) = K_i \bar{x}_i(t)$ such that $\bar{A}_i \triangleq A_i + B_i K_i$ is stable.

The basic idea of receding horizon control (RHC) is: at each time instant t_k, the state $x(t_k)$ is sampled and an optimization problem is solved to generate a optimal control sequence $u^*(s; t_k)$, where $s \in [t_k, t_k + T]$ and T is the prediction horizon; the subsequence $u^*(s; t_k)$, $s \in [t_k, t_k + \delta]$ is applied to the system as control input until the next sampling time instant $t_{k+1} = t_k + \delta$, where $\delta < T$ is the sampling period. To ensure closed-loop stability and feasibility, a conventional technique is to force the terminal state $x(t_k + T)$ in the terminal set at each time instant. By doing so, one can construct a feasible control trajectory to prove feasibility and further prove the optimal cost function (i.e., Lyapunov function candidate) decreasing, leading to closed-loop stability. For more details, see, e.g., [22]. To design the terminal set, a well-known result from receding horizon control (RHC) [22,23,21] is recalled.

Lemma 1. *For the nominal system of (1), suppose that Assumptions 1 and 2 hold. Given a stabilizable K_i, and two symmetric positive-definite matrices Q_i and R_i, there exist a constant $\varepsilon_i > 0$ and a matrix $P_i > 0$, such that: if $\bar{x}_i(t) \in \Omega_i(\varepsilon_i)$, then (1) $V_i(\bar{x}_i(t))$ is qualified as a Lyapunov function for the system $\dot{\bar{x}}_i(t) = f_i(\bar{x}_i(t), K_i \bar{x}_i(t))$, and $\dot{V}_i(\bar{x}_i(t)) \leqslant -\|\bar{x}_i(t)\|^2_{Q_i^*}$, (2) $u_i(t) = K_i \bar{x}_i(t) \in \mathcal{U}_i$, where $V_i(\bar{x}_i(t)) \triangleq \|\bar{x}_i(t)\|^2_{P_i}$, $\Omega_i(\varepsilon_i) \triangleq \{\bar{x}_i(t) : V_i(\bar{x}_i(t)) \leqslant \varepsilon_i^2\}$ and it is called the terminal set, and $Q_i^* = Q_i + K_i^{\mathrm{T}} R_i K_i$.*

3. Periodic event-triggered DRHC

3.1. Periodic event-triggering in optimization

For the overall system, define the sequence $\{t_k, k \in \mathbb{N}\}$ as the time instants when triggering conditions are checked, where $t_{k+1} - t_k = \tau > 0$, and τ is called the testing period. Due to the event-triggered strategy, the information update of overall system will possibly become asynchronous. To characterize individual behavior of each agent i, define the sequence $\{t_k^i, k \in \mathbb{N}\}$ as the time instants at which the optimization problem is solved and the information is sent out, where the subscript i represents the index of agent, and subscript k denotes by the time instant. Furthermore, the prediction horizon of the optimization problem also needs to adapt to the periodic event-triggered strategy. Here, the prediction horizon is set as $T = n_0 \tau$, where $n_0 \in \mathbb{N}$ and $n_0 > 1$. n_0 is called the testing length, and is the maximum times of checking the triggering conditions in each round of optimization. That is, if the triggering condition is checked continuously by n_0 times starting from t_k^i, and it is still not met, then the optimization problem will be solved automatically again at $t_k^i + n_0 \tau$. As a result, $t_{k+1}^i - t_k^i = n\tau$, where n might take any integer between 1 and n_0. In the rest of this paper, three types of control input trajectories will be utilized. For the ease of presentation, we summarize the notations of them here.

- $\hat{u}_i(s; t_k^i)$ denotes by the predicted control input trajectory for agent i at time instant t_k^i, which is a variable for optimization problems, where $s \in [t_k^i, t_k^i + l]$ for some $0 \leqslant l < \infty$;
- $\hat{u}_i^a(s; t_k^i)$ denotes by the assumed control input trajectory for agent i at time instant t_k^i, which is a fixed trajectory and is