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Generalized (complete, lag, anticipated) synchronization of discrete-time chaotic systems

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Abstract

A unify definition of generalized (complete, lag, anticipated) synchronization in discrete-time chaotic systems is proposed in this paper. Based on the contraction mapping theorem, a new general scheme is proposed for the generalized synchronization of discrete-time chaotic and hyper-chaotic systems. The well-known Hénon mapping and generalized hyper-chaotic Hénon mapping are chosen to illustrate the proposed scheme. Numerical simulations are also shown to verify the effectiveness of the proposed control method.

Keywords: Generalized chaos synchronization; Discrete-time chaotic systems; Contraction mapping

1. Introduction

Over the past decade, chaos synchronization has received increasing interest attention since the pioneer work of Fujisaka and Yamada [1], Pecora and Carroll [2]. Many special issues on the control and synchronization of chaos have been published [3–8] due to its possible application in various fields, such as application to control theory, secure communication, chemical reaction and encoding message [9].

Up to now, many types of synchronization have been proposed in dynamical systems such as complete synchronization (CS), generalized synchronization (GS), frequency synchronization (FS), lag synchronization (LS), anticipated synchronization (AS), phase synchronization (PS), anti-phase synchronization (APS), Q–S synchronization, etc. Recently, a general definition of synchronization was presented by Brown and Kolarev (BK) which included CS, LS, PS, and FS [10]. Subsequently, Boccaleti et al. [11] developed a unifying framework of synchronization of coupled dynamical systems which generalized the BK-type synchronization to a more condensed and concrete form than the BK-type synchronization. In the last decade, many schemes have been proposed for chaos synchronization, such as Pecora and Carroll method [2], backstepping design method [12], impulse control method [13], one-way coupling method [14], adaptive control [15–17], active control [18,19], invariant manifold method [20], time-delay feedback approach [21], contraction mapping method [22], OGY method [23], etc.

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In this paper, we shall discuss a type of generalized (complete, lag, anticipated) synchronization in discrete-time chaotic systems based on the contraction mapping. The layout of the paper is as follows. In Section 2, the definitions of generalized synchronization and the contraction mapping are presented. The synchronization control scheme of discrete-time chaotic systems is presented in Section 3. In Section 4, the scheme is applied to investigate the generalized synchronization between the identical Hénon mapping and the generalized Hénon mapping, and numerical simulations are also given to verify the effectiveness of the proposed scheme. A conclusion ends the paper.

Throughout this paper, $\|A\|_F$ denotes the Frobenius norm of $m \times n$ matrix A, i.e., $\|A\|_F = \left(\sum_{i=1}^{m}\sum_{j=1}^{n}|a_{ij}|^2\right)^{1/2}$ [24] .

2. Preliminaries

In this section, we'll give the definition of generalized (complete, lag, anticipated) synchronization and contraction mapping which will be used later. Assumed that the two discrete-time chaotic systems: the drive (master) system and response (slave) system are defined, respectively, by the following equations:

$$x(k+1) = f(x(k)) \tag{1}$$

$$y(k+1) = f(y(k)) + U(x, y, k) \tag{2}$$

where $x(k), y(k) \in R^n$ are the state vectors of the drive system and response system at time k, f is a mapping from R^n to itself, and U is the controller to be determined for the purpose of synchronizing the two chaotic systems.

Definition 1. The drive system (1) and response system (2) are said to be (complete, lag, anticipated) synchronization while $\lim_{k\to\infty}\|y(k) - x(k-\tau)\|_F = 0$ $(\tau \in Z)$, which include

- complete synchronization (CS) while $\tau = 0$;
- lag synchronization (LS) while $\tau > 0$;
- anticipated synchronization (AS) while $\tau < 0$.

Definition 2. It is said that the drive system (1) and response system (2) are globally

- generalized complete synchronization (GCS) while $\tau = 0$;
- generalized lag synchronization (GLS) while $\tau > 0$;
- generalized anticipated synchronization (GAS) $\tau < 0$;

with respect to the matrix M, if there exist a controller $U(x, y, k)$ and a $n \times n$ matrix M such that all trajectories $(x(k-\tau), y(k))$ with any initial conditions $(x(0), y(0))$ approach the manifold $K = \{(x(k-\tau), y(k)) : y(k) = Mx(k-\tau)\}$ as time k goes to infinity, that is to say $\lim_{k\to\infty}\|e(k)\|_F = \lim_{k\to\infty}\|y(k) - Mx(k-\tau)\|_F = 0$.

Definition 3 [25]. Let E be a Banach space with the norm $\|\cdot\|$, and F be a mapping of E onto itself. Then F is said to be a contraction mapping if their exists a constant $\alpha_F (0 \leqslant \alpha_F < 1)$ such that the inequality $\|F(x) - F(y)\| \leqslant \alpha_F\|x - y\|$ holds for every pair of points $x, y \in E$, where α_F is called a contraction constant of F on E.

3. Generalized synchronization scheme

We assume that mapping f in (1) and (2) can be rewritten as $f = g + h$, where both g and h are mappings from R^n to itself , and g is a contraction mapping on a closed set $E \subseteq R^n$. Let $\alpha_g (0 \leqslant \alpha_g < 1)$ be the contraction constant of g on E. Then the drive system and response system should be

$$x(k+1) = g(x(k)) + h(x(k)) \tag{3}$$

$$y(k+1) = g(y(k)) + h(y(k)) + U(x, y, k) \tag{4}$$