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Designing of Run Rules t control charts for monitoring changes in the process mean

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ABSTRACT

In the literature, the Shewhart t control chart has already been investigated and used to detect shifts in a process; anyway, it has a poor statistical sensitivity in the detection of small or moderate process shifts. In this work, new Phase II Run Rules t control charts are proposed, and their parameters are determined. A Markov chain methodology is used to evaluate the statistical performance of these charts. We provide an extensive numerical analysis consisting of several tables and figures to discuss the statistical performance of the proposed charts for several scenarios. The obtained results show that the proposed control charts are more sensitive to process shifts than Shewhart t control chart and robust against errors in errors in estimating the process standard deviation or changing standard deviation. Finally, an illustrative example from the food industry is provided for illustration.

1. Introduction

Statistical Process Control (SPC) is a technique for applying statistical analysis to measure, monitor and control processes in order to achieve process stability and to improve capability through the reduction of the process variability, see Montgomery [1]. Among all the methods in SPC, control charts play a very important role. It is important to note that the goals of one-class classification methods and control charts are similar since only one class is represented in the training data which can be used to learn its characteristics as well as to provide a measure of abnormal behavior of new observations. They are several kinds of control charts. The Shewhart [2] type control charts are popular, easy to implement, efficient to detect large changes in a process but rather inefficient in detecting small or moderate changes. For this reason, several methods have been proposed in SPC literature to overcome this problem. In recent years, many researchers focus on developing advanced control charts with various applications in manufacturing and service processes, for example, see Nezhad and Niaki [3], Castagliola et al. [4], Tran et al. [5], Tran et al. [6] and Kang and Kim [7]. Among these methods, the implementation of memory-type control charts like the exponentially weighted moving average (EWMA) or cumulative sum (CUSUM) charts is particularly efficient but their relatively complexity of use has not been widely accepted everywhere by the quality practitioners, see Tran [8].

It is known that, the performance of a $\overline{X}$ type control chart is usually evaluated under the assumption that there are no errors in estimating the process standard deviation or changing standard deviation. This assumption is not always the case in practice. Therefore, Zhang et al. [9] developed the Shewhart-t and EWMA t control charts which are more robust to shifts occurring to the process standard deviation than the $\overline{X}$ type control charts. Then, the properties and design of t type control charts have been thoroughly investigated by many authors. For further details see, for instance, Celano et al. [10], Celano et al. [11] Celano et al. [12], Castagliola et al. [13], Sitt et al. [14] to name a few.

In practice, Shewhart type control charts are known to be rather inefficient in detecting small or moderate process shifts. The statistical sensitivity of a Shewhart control chart can be improved by using supplementary Run Rules. The most popular Run Rules were suggested by Western-Electric [15]. Run Rules have also been suggested by Page [16], Roberts [17], Bissell [18], and Wheeler [19]. Then, Champ and Woodall [20] obtained the exact formula to evaluate the Run-Length distribution by using a Markov chain approach for the Shewhart control chart with supplementary Run Rules. Other studies that have investigated the Shewhart charts with Run Rules were reported in Palm [21], Divoky and Taylor [22], Lowry et al. [23], Champ and Woodall [24], Klein [25],

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Shmueli and Cohen [26], Fu et al. [27], Khoo [28], Zhang and Castagliola [29], Tran et al. [30], Tran [8] and Tran [31].

In this paper, we introduce Run Rules t control charts (denoted as $RR_{r,s}-t$ control charts). The rest of this paper proceeds as follows: in Section 2 the $RR_{r,s}-t$ control charts are defined; in Section 3 the main run length properties of the $RR_{r,s}-t$ control chart are obtained by using a dedicated Markov chain model; in Section 4, the design parameters and the performance of the $RR_{r,s}-t$ control charts are provided for different scenarios and a comparison is made with Shewhart-t control chart already proposed in literature; in Section 5, the robustness of the $RR_{r,s}-t$ control charts is investigated; in Section 6 an illustrating example showing the implementation of the $RR_{r,s}-t$ control charts are provided and, finally, some concluding remarks and recommendations are made in Section 7.

2. Implementation of the $RR_{r,s}-t$ control charts

Let $\{X_{i,1},\ldots,X_{i,n}\}$, at time $i=1,2,\ldots$, be a Phase II sample of n independent normal random variables, i.e., $X_{i,j}\sim(\mu_0+a\sigma_0, b\sigma_0)$ random variables, where μ_0 and σ_0 are the nominal mean and standard-deviation, respectively, both assumed known, while a and b are the standardized mean and standardized deviation shifts. The process has shifted if the process mean μ_0 and/or the process standard deviation σ_0 have changed ($a\neq 0$ and/or $b\neq 1$).

Let $\overline{X}_i$ and S_i be the sample mean and the sample standard-deviation of subgroup $\{X_{i,1},\ldots,X_{i,n}\}$, at time i, i.e.,

$$\overline{X}_i=\frac{1}{n}\sum_{i=1}^{n}X_i,$$

and

$$S_i=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i-\overline{X}\right)^2}.$$

Then, statistic T_i defined by

$$T_i=\frac{\overline{X}_i-\mu_0}{S_i/\sqrt{n}}, i=1,2,\ldots. \tag{1}$$

It is well known that the moments of the t distribution cannot be defined. For this reason, Zhang et al. [9] suggested a two-sided Shewhart-type chart for monitoring T_i with probability-type control limits UCL_t and LCL_t defined as

$$\begin{aligned}UCL_t&=F_t^{-1}\left(1-\frac{\alpha}{2}\middle|n-1\right),\\ LCL_t&=-UCL_t,\end{aligned} \tag{2}$$

where $F_t^{-1}(\cdot|n-1)$ is the inverse distribution function of the Student's t-distribution with $n-1$ degrees of freedom, α is the false alarm rate.

The goal of this paper is to define and compare several Run Rules strategies to monitor T_i. It is important to note that, like in Klein [25], we only focus on *pure* Run Rules type charts which only require a single couple of limits and assume that an out-of-control condition must be signaled only if the selected Run Rules pattern occurs. As shown in Klein [25], the advantage of pure Run Rules type charts (compared to *composite* ones, requiring control as well as warning limits) is the simplicity of their implementation and interpretation, see Tran et al. [30] for more details. More specifically, in this paper, we will only focus on the 2-out-of-3, 3-out-of-4 and 4-out-of-5 t control charts (from now on, denoted as $RR_{r,s}-t$ chart) which correspond to the case $(r=2,s=3)$ and $(r=3,s=4)$. In the r-out-of-s Run Rules, an out-of-control signal is obtained if r out of s successive values t_i are plotted above an upper control limit $UCL_{RR_{r,s}-t}$ or r out of s successive points are plotted below a lower control limit $LCL_{RR_{r,s}-t}$ with

$$\begin{aligned}UCL_{RR_{r,s}-t}&=F_t^{-1}\left(1-\frac{\alpha_r}{2}\middle|n-1\right),\\ LCL_{RR_{r,s}-t}&=-UCL_{RR_{r,s}-t},\end{aligned} \tag{3}$$

where $\alpha_r>0$ is a chart parameter to be defined.

3. Run length properties of the $RR_{r,s}-t$ charts

Concerning the $RR_{2,3}-t$ control chart ($r=2,s=3$), the sequence of points plotted on such charts can be modelled as a stochastic process and the statistical properties of these control charts can be obtained by using the following Markov Chain matrix $\mathbf{P}$, where the 8th state corresponds to the absorbing state:

$$\mathbf{P}=\begin{pmatrix}\mathbf{Q} & \mathbf{r}\\ \mathbf{0}^T & 1\end{pmatrix}=\left(\begin{array}{ccccccc|c} 0 & 0 & 0 & p_C & p_L & 0 & 0 & p_U\\ 0 & 0 & 0 & 0 & 0 & 0 & p_C & p_L+p_U\\ p_C & p_L & 0 & 0 & 0 & 0 & 0 & p_U\\ 0 & 0 & p_U & p_C & p_L & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & p_U & p_C & p_L\\ p_C & 0 & 0 & 0 & 0 & 0 & 0 & p_L+p_U\\ 0 & 0 & p_U & p_C & 0 & 0 & 0 & p_L\\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{array}\right),$$

where $\mathbf{0}=(0,0,\ldots,0)^T$, $\mathbf{Q}$ is the $(7,7)$ matrix of transient probabilities, the $(7,1)$ vector $\mathbf{r}$ satisfies $\mathbf{r}=\mathbf{1}-\mathbf{Q}\mathbf{1}$ (i.e. row probabilities must sum to 1), with $\mathbf{1}=(1,1,1,1,1,1,1)^T$. The corresponding $(7,1)$ vector $\mathbf{q}$ of initial probabilities associated with the transient states is equal to $\mathbf{q}=(0,0,0,1,0,0,0)^T$ (i.e. the initial state is the fourth one).

The proposed Markov chain model for $RR_{r,s}-t$ control chart can be extended to "longer" Run Rules like, for instance, the 3-out-of-4 and the 4-out-of-5 Run Rules. For the $RR_{r,s}-t$ chart the matrix $\mathbf{Q}_{(25\times 25)}$ of transient probabilities is equal to