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Model predictive control suitable for closed-loop re-identification

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HIGHLIGHTS

- A new MPC suitable for closed-loop re-identification is proposed.
- A re-identification needs to be developed in a closed-loop fashion, since the process cannot be stopped.
- The main problem is the conflict between the control and identification objectives.
- A generalization, from punctual stability to (invariant) set stability, is done to avoid the conflict.
- The proposal could be potentially applied to real processes.

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ABSTRACT

The main problem of a closed-loop re-identification procedure is that, in general, the dynamic control and identification objectives are conflicting. In fact, to perform a suitable identification, a persistent excitation of the system is needed, while the control objective is to stabilize the system at a given equilibrium point. However, a generalization of the concept of stability, from punctual stability to (invariant) set stability, allows for a flexibility that can be used to avoid the conflict between these objectives. Taking into account that an invariant target set includes not only a stationary component, but also a transient one, the system could be excited without deteriorating the stability of the closed-loop. In this work, a MPC controller is proposed that ensures the stability of invariant sets at the same time that a signal suitable for closed-loop re-identification is generated. Several simulation results show the propose controller formulation properties.

1. Introduction

Model predictive control (MPC) is typically implemented as a lower stage of a hierarchical control structure. The upper level stages are devoted to compute, by means of a stationary optimization, the targets that the dynamic control stage (MPC) should reach to economically optimize the operation of the process. Since both the dynamic and stationary optimizations are model-based optimizations, a periodic updating of the model parameters are desired to reach meaningful optimums. In this context, a re-identification procedure should be developed in a closed-loop fashion, since the process cannot be stopped each time an update is needed. As it is known, the main problem of a closed-loop identification is that the dynamic control objectives are incompatible with the identification objectives [1]. In fact, to perform a suitable identification, a persistent excitation of the system modes is needed, while the controller takes this excitation as disturbance and tries to reject this disturbance to stabilize the system.

From a general point of view, the closed-loop identification methods fall into the following main groups [2]. The direct approach ignores the feedback law and identifies the open-loop system using measurements of the input and the output. The indirect approach identifies the closed-loop transfer function and determines the open-loop parameters subtracting the controller dynamic. To do that, the controller dynamics must be linear and known. The joint input–output approach takes the input and output jointly, as the output of a system produced by some extra input or set-point signal. Since the last two methods need the exact knowledge of a linear controller, they are not directly applicable for closed-loops under constrained MPC controllers.

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Several strategies were developed to perform closed-loop re-identification under MPC controllers: [3] proposed a controller named Model Predictive Control and Identification (MPCI) where a persistent excitation condition is added by means of an additional constraint in the optimization problem. This strategy, which was explored later in [4], turns the MPC optimization problem non-convex, and so, most of the well-known properties of the MPC formulation cannot be established. [5] proposed a strategy that manipulates the steady-state target optimization (in the hierarchical MPC control structure) in order to excite the system. In the context of data-driven MPC formulations (i.e., MPC that are designed to perform predictions directly from collected data), the subspace identification method is exclusively used [6]. In [7–9] several approaches are presented, where a closed-loop re-identification is needed to update the data for predictions. Though preliminary studies were made according to the trade-off between stability and excitation, no definitive results were presented.

In general, none of the reports cited in this section have shown results regarding the system stability of the MPC while the system is being re-identified. In this work, based on the concept of stability of an invariant set (as a generalization of stability of a point), a MPC controller with a extended domain of attraction is proposed, which ensures stability at the same time that a persistent excitation can be generated to perform a closed-loop re-identification. Some preliminary results regarding the complete strategy presented in this work were recently presented in [10].

Notation. Matrices $I_n \in \mathbb{R}^{n\times n}$ and $0_{n,m} \in \mathbb{R}^{n\times m}$ denote the identity matrix and the null matrix, respectively. A *C-set* is a convex and compact set that contains the origin. A *proper C-set* is a C-set that contains the origin as an interior point. Consider two sets $\mathcal{U} \subseteq \mathbb{R}^n$ and $\mathcal{V} \subseteq \mathbb{R}^n$, containing the origin and a real number λ. The Minkowski sum $\mathcal{U} \oplus \mathcal{V} \subseteq \mathbb{R}^n$ is defined by $\mathcal{U} \oplus \mathcal{V} = \{(u+v) : u \in \mathcal{U}, v \in \mathcal{V}\}$; the set $(\mathcal{U} \setminus \mathcal{V}) \subseteq \mathbb{R}^n$ is defined as $\mathcal{U} \setminus \mathcal{V} = \{u : u \in \mathcal{U} \wedge u \notin \mathcal{V}\}$; and the set $\lambda\mathcal{U} = \{\lambda u : u \in \mathcal{U}\}$ is a scaled set of $\mathcal{U}$. $|v|_{\mathcal{V}}$ is the distance from v to $\mathcal{V}$. The boundary of a set $\mathcal{U}$ is defined as $\partial\mathcal{U}$. Given a continuous function $\Psi : \mathbb{R}^n \to \mathbb{R}$, and $\gamma \geq 0$, the level set $\mathcal{N}[\Psi, \gamma]$ is defined by $\mathcal{N}[\Psi, \gamma] = \{x : \Psi(x) \leq \gamma\}$. $\mathbb{I}_{m:n}$ denotes the nonnegative integers from m to n. Given $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$, $\|x-y\|_M^2 = (x-y)^T M(x-y)$, with $M \in \mathbb{R}^{n\times n}$.

2. Problem statement and preliminaries

Consider a system described by a linear time-invariant discrete-time model

$$x^+ = Ax + Bu, \qquad y = Cx,$$

where $x \in \mathbb{R}^n$ is the system state, x^+ is the successor state, $u \in \mathbb{R}^m$ is the current control, and $y \in \mathbb{R}^p$ is the system output. The state, the control input and the output at discrete-time instant k are denoted as $x(k)$, $u(k)$ and $y(k)$, respectively. The system is subject to hard constraints on state and input, $(x(k), u(k)) \in \mathcal{Z} \triangleq (\mathcal{X} \times \mathcal{U}) \subset \mathbb{R}^{n+m}$ for all $k \geq 0$, where $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{U} \subset \mathbb{R}^m$. Furthermore, the following assumption holds:

Assumption 1. Matrix A has all its eigenvalues strictly inside the unit circle, the pair (A, B) is controllable and the state (corresponding to the true plant) is measured at each discrete-time instant. Furthermore, the set $\mathcal{X}$ is convex and closed, the set $\mathcal{U}$ is convex and compact and both contain the origin in their interior. For simplicity, $A\mathcal{X} \subseteq \lambda\mathcal{X}$, with $\lambda \in [0, 1)$.

Previous to the controller formulation, some necessary definitions helpful to generalize the concepts of equilibrium and invariance are introduced. To simplify the notation, we denote system $x^+ = Ax + Bu$, $(x, u) \in \mathcal{Z}$ as *Non-autonomous system* ($\mathcal{N}_{sys}$) and system $x^+ = Ax + B\kappa(x)$, $(x, \kappa(x)) \in \mathcal{Z}$, where $\kappa(x)$ is a state feedback, as the *Controlled system* ($\mathcal{C}_{sys}$). Accordingly, for a given sequence of control inputs, $\mathbf{u} = \{u(0), \ldots, u(j-1)\}$ and a given initial state $x(0) = x$, the solution of $\mathcal{N}_{sys}$ will be denoted as: $x(j) = \phi(j; x, \mathbf{u}) = A^j x(0) + \sum_{i=0}^{j-1} A^{j-i-1}Bu(i)$, $j \in \mathbb{I}_{\geq 1}$. Similarly, for a given initial state $x(0) = x$, the solution of $\mathcal{C}_{sys}$ will be denoted as: $x(j) = \phi_\kappa(j; x) = A^j x(0) + \sum_{i=0}^{j-1} A^{j-i-1}B\kappa x(i)$, $j \in \mathbb{I}_{\geq 1}$, for $j \in \mathbb{I}_{\geq 1}$.

Definition 1 (*Control Equilibrium Set—CES*). A set $\Omega \subseteq \mathcal{X}$ is a control equilibrium set for $\mathcal{N}_{sys}$, if for every point $x \in \Omega$ the condition $x^+ = x$ holds for some $u \in \mathcal{U}$.

The *maximal* CES, $\mathcal{X}_{ss}$, is given by $\mathcal{X}_{ss} = (GB\mathcal{U}) \cap \mathcal{X}$, where $G = (I_n - A)^{-1}$. In the case of controlled systems, $\mathcal{C}_{sys}$, we simply say that a control equilibrium set Ω is an *equilibrium set—(ES)*, with $u = \kappa(x)$. The proper generalization of the concept of equilibrium point is not the concept of equilibrium set, as a mere aggregation of steady-state points, but the concept of invariant set (associated to an equilibrium set), in the sense that both the equilibrium point and the invariant set are geometric entities such that, if the system reaches them, it remains in them indefinitely [11–13]:

Definition 2 (λ*-Control Invariant Set—*λ*-CIS*). A proper C-set $\Omega \subseteq \mathcal{X}$ is λ-control invariant, with $\lambda \in [0, 1]$, for $\mathcal{N}_{sys}$, if $x \in \Omega$ implies $x^+ \in \lambda\Omega$, for some $u \in \mathcal{U}$.

Again, in the case of controlled systems, $\mathcal{C}_{sys}$, a λ-Control Invariant set is simply a λ*-Invariant Set—*(λ*-IS*), with $u = \kappa(x)$. Furthermore, if $\lambda = 1$, the sets are simply *Invariant sets*, and if $\lambda \in [0, 1)$, the sets are known as *Contractive sets*. The concept of invariant set, as a generalization of an equilibrium point, makes possible the generalization of the concept of attractivity of an equilibrium point. Then, we can define the attractivity of an IS set as follows [14]:

Definition 3 (*Local Attractivity of An IS Set*). The IS set $\Omega \subset \mathcal{X}$ is locally attractive for $\mathcal{C}_{sys}$ if for each x in a vicinity of Ω (that we call the domain of attraction), it follows that $|\phi_\kappa(j; x)|_\Omega \to 0$, $\phi_\kappa(j; x) \in \mathcal{X}$, $\kappa(\phi_\kappa(j; x)) \in \mathcal{U}$ as $j \to \infty$.

3. Target invariant set for identification

The objective of this section is to propose a set (in the state space) that is *invariant* under the excitation procedure necessary to perform a suitable identification and, at the same time, can be used as the *attractive target* set (generalized equilibrium) by an MPC controller. As known, to estimate a model from measured input and output data, each (controllable) mode of the system must be excited. To do that, the excitation input signal must contain enough variability. This property is generally indicated by the notion of *persistence of excitation* [15]. The persistent excitation input sequences might be of several forms, going from a Pseudo-Random Binary Signal (PRBS) to a Filtered Pseudo Gaussian White Noise Signal. A recent formulation proposed a filtered Gaussian inputs signal specifically designed for MPC [16]. Independently of the form, the persistent excitation sequences have two main properties: they are bounded, belonging to a compact set smaller than $\mathcal{U}$, and more subtle, they have a persistent-variability behavior. Regarding the first property, we define:

Definition 4 (*Excitation Input Set, EIS*). An input proper C-set $\mathcal{U}^t \subset \mathcal{U} \subset \mathbb{R}^m$, with enough size to excite the system will be denoted as the excitation input set.

The set $\mathcal{U}^t$ defines a *class* of sequences $\mathbf{u} = \{u(0), \ldots, u(T_{id}-1)\}$ – denoted by $\mathcal{C}_{\mathcal{U}^t}$ – such that $u(i) \in \mathcal{U}^t$ for $i \in \mathbb{I}_{0:T_{id}-1}$,