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Short communication

Bifurcation continuation, chaos and chaos control in nonlinear Bloch system

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Abstract

A detailed analysis is undertaken to explore the stability and bifurcation pattern of the nonlinear Bloch equation known to govern the dynamics of an ensemble of spins, controlling the basic process of nuclear magnetic resonance. After the initial analysis of the parameter space and stability region identification, we utilize the MATCONT package to analyze the detailed bifurcation scenario as the two important physical parameters γ (the normalized gain) and c (the phase of the feedback field) are varied. A variety of patterns are revealed not studied ever before. Next we explore the structure of the chaotic attractor and how the identification of unstable periodic orbit (UPO) can be utilized to control the onset of chaos.

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1. Introduction

The dynamics of an ensemble of spins in an external magnetic field is of utmost importance in the understanding various phenomena related to the extremely important field for nuclear magnetic resonance (NMR). The actual physical process can be described as a combination of the precession of the spins about a magnetic field and damping of transverse and longitudinal component of magnetization with different relaxation time Γ_1 and Γ_2. Manifestations of nonlinear spin dynamics could be observed due to the presence of an additional field which is proportional to the components of the magnetization. The dipolar magnetizing field could be shown to give rise to multiple echoes in liquid helium [1] and in water at high magnetic field [2]. Moreover, the effect of radiation damping or demagnetizing field [3–5] can result in the existence of pseudo-multiquanta

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peaks. A good description for the whole phenomena is given by Bloch equation, whose preliminary analysis in very special cases were reported by Abergel et al. [6], who studied some special situation when the relaxation time was large or small in an asymptotic way. Though they obtained the chaotic attractor in the general case, the over all scenario and the various channel of bifurcation was not obtained. Here in this communication we have studied in detail the stability zones in parameter space and the different route of continuation of the associated bifurcation with the help of the software package MATCONT [7–9], revealing a rich structure of dynamical transitions. In the next part of our paper have studied the structure of the unstable periodic orbits and have devised a method to control the chaos so generated.

The work of OGY [10] and Pecora and Carroll [11] led to wide applications outside the traditional scope of chaos and nonlinear dynamics. The unified study of chaos control and synchronization was carried out in [12,13]. Different approaches have been suggested for such procedure [14,15]. Incidentally Yu et al. [16] proposed a method for controlling chaos in the form of special nonlinear feedback. The validity of this method based on the stability criterion of linear system and can be called stability criterion method (SC method). The construction of a nonlinear form of limit continuous perturbation feedback by a suitable separation of the system in the SC method does not change the form of the desired UPO. The closed return pair technique [17] is utilized to estimate the desired periodic orbit chosen from numerous UPO's embedded within a chaotic attractor. The advantage of this method is that the effect of the control can be generalized directly without calculation of the maximal lyapunov exponent of the UPO using the linearization of the system. The method has been used by researchers in the control of Rössler system, chaotic altitude motion of a spacecraft and the control of two coupled Duffing oscillators.

In this communication, the local stability analysis of the nonlinear Bloch equations in a particular parameter regime has been discussed and then the different route of continuation of the associated bifurcation analysis is investigated. Finally, the chaotic scenario of the system is controlled using the stability criterion method which involves the construction of input perturbation function.

2. Formulation

The dynamics of an ensemble of spins usually described by the nonlinear Bloch equation is very important for the understanding of the underlying physical process of nuclear magnetic resonance. The basic process can be viewed as the combination of a precession about a magnetic field and of a relaxation process, which gives rise to the damping of the transverse component of the magnetization with a different time constant. Introduction of an externally generated feedback field was first proposed by Hobson and Kayser [18] some decades ago in the context of CW NMR spectroscopy, where the effect of the line shape was emphasized. In the present communication, we adopt a dynamical system view point and focus on the time evolution of the magnetization, with the normalized gain and the phase of the feedback field thought of as basic parameters. In this context it may be mentioned that some relevant work related to the onset of chaos was done earlier by Abergel [19]. But one should note that detailed scenario of bifurcation pattern was never examined, though various anomalies that arise in NMR experiments have been studied in terms of chaos theory. Uçar et al. [20] extended the calculation of Abergel and demonstrated the synchronization of two such Bloch systems through 'Master' and 'Slave' arrangement through a suitably designed controller.

The basic model is derived from a magnetization $\mathbf{M}$ precessing in the magnetic induction field $\mathbf{B}_0$ in the presence of a constant radiofrequency field $\mathbf{B}_1$ with intensity $\mathbf{B}_1 = \frac{\omega_1}{\gamma}$ and frequency ω_{rf}. The following modified nonlinear Bloch equation govern the evolution of the magnetization,

$$\dot{x} = \delta y + \gamma z(x\sin(c) - y\cos(c)) - \frac{x}{\Gamma_2}, \tag{2.1}$$

$$\dot{y} = -\delta x - z + \gamma z(x\cos(c) + y\sin(c)) - \frac{y}{\Gamma_2}, \tag{2.2}$$

$$\dot{z} = y - \gamma\sin(c)(x^2 + y^2) - \frac{(z-1)}{\Gamma_1}, \tag{2.3}$$

where the variables are properly scaled [6]. This differential equations (2.1)–(2.3) cannot be solved analytically in the general case. So one has to resort to numerical computation to study the time evolution of the magne-