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Solitary wave solutions of the modified equal width wave equation

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Abstract

In this paper we use a linearized numerical scheme based on finite difference method to obtain solitary wave solutions of the one-dimensional modified equal width (MEW) equation. Two test problems including the motion of a single solitary wave and the interaction of two solitary waves are solved to demonstrate the efficiency of the proposed numerical scheme. The obtained results show that the proposed scheme is an accurate and efficient numerical technique in the case of small space and time steps. A stability analysis of the scheme is also investigated.

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Keywords: MEW equation; Linearized implicit finite-difference; Solitary waves

1. Introduction

The one-dimensional generalized equal width (GEW) equation is of the form

$$\frac{\partial U}{\partial t} + \varepsilon U^m \frac{\partial U}{\partial x} - \mu \frac{\partial}{\partial t}\left(\frac{\partial^2 U}{\partial x^2}\right) = 0 \tag{1}$$

with the physical boundary conditions $U \to 0$ as $x \to \pm\infty$, where t is time, x is the space coordinate, $U(x,t)$ is the wave amplitude, and ε, μ and m are positive parameters. This equation has a solitary wave solution of the form [13]

$$U(x,t) = \left[\frac{c(m+1)(m+2)}{2\varepsilon}\operatorname{sech}^2\left(\frac{m}{2\sqrt{\mu}}(x - ct - x_0)\right)\right]^{1/m}. \tag{2}$$

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In Eq. (1) when $m=1$ we get equal width (EW) equation introduced by Morrison et al. [1]

$$\frac{\partial U}{\partial t}+\varepsilon U\frac{\partial U}{\partial x}-\mu\frac{\partial}{\partial t}\left(\frac{\partial^2 U}{\partial x^2}\right)=0, \tag{3}$$

which plays a major role in the study of nonlinear dispersive waves since it describes a broad class of physical phenomena such as shallow water waves and ion acoustic plasma waves [2,3]. Analytical solutions of the EW equation are known with only a restricted set of boundary and initial conditions. Therefore, many numerical methods [4–10] have been used for solving the EW Equation (3) with various boundary and initial conditions.

In Eq. (1) when $m=2$ we get the following modified equal width (MEW) equation to be considered in this paper:

$$\frac{\partial U}{\partial t}+\varepsilon U^2\frac{\partial U}{\partial x}-\mu\frac{\partial}{\partial t}\left(\frac{\partial^2 U}{\partial x^2}\right)=0. \tag{4}$$

Like the EW equation, the MEW equation with a limited set of boundary and initial conditions has an analytical solution. Therefore, many authors have used various kinds of numerical methods to solve Eq. (4). Zaki [11] used a quintic B-spline collocation method to investigate the motion of a single solitary wave, interaction of two solitary waves and birth of solitons for the MEW equation. Hamdi et al. [12] derived exact solitary wave solutions of the GEW Eq. (1). Evans and Raslan [13] solved the GEW equation by a collocation finite element method using quadratic B-splines to obtain the numerical solutions of the single solitary wave, solitary waves interaction and birth of solitons. Wazwaz [14] investigated the MEW equation and two of its variants by the tanh and the sine–cosine methods.

In this paper we have used a linearized implicit finite difference method in solving the MEW equation. The performance and accuracy of the method has been tested on two numerical experiments of wave propagation: the motion of a single solitary wave and the interaction of two solitary waves.

2. Linearized implicit finite difference scheme

For the numerical treatment, the spatial variable x of the problem is restricted over the finite region $a\leqslant x\leqslant b$. The solution domain $a\leqslant x\leqslant b$, $t\geqslant 0$ is divided into intervals Δx in the direction of the spatial variable x and Δt in the direction of time t such that $x_i=a+i\Delta x$, $i=0,1,\ldots,N$ $(N\Delta x=b-a)$; $t_j=j\Delta t$, $j=0,1,\ldots,J$, and the numerical solution of U at the grid point $(i\Delta x, j\Delta t)$ is denoted by $U_{i,j}$.

In the finite difference method, the dependent variable and its derivatives are approximated by the finite difference approximation. This approximation will lead to either a single explicit equation or a system of difference equations. Applying the classical implicit finite-difference method to non-linear problems normally give non-linear system of equations which cannot be solved directly.

Eq. (4) can be written as

$$\frac{\partial U}{\partial t}+\frac{\varepsilon}{3}\frac{\partial U^3}{\partial x}-\mu\frac{\partial}{\partial t}\left(\frac{\partial^2 U}{\partial x^2}\right)=0. \tag{5}$$

Using the forward difference approximation for $\partial U/\partial t$, the Crank–Nicolson difference approximation for $\partial U^3/\partial x$, the central difference approximation for $\partial^2 U/\partial x^2$ and then utilizing the central difference operator δ defined by $\delta_x U_{i,j}=U_{i+1,j}-U_{i-1,j}$ (see, e.g. [15]), Eq. (5) yields the system of algebraic equations

$$\frac{U_{i,j+1}-U_{i,j}}{\Delta t}+\frac{\varepsilon}{12\Delta x}\{\delta_x(U^3_{i,j+1})+\delta_x(U^3_{i,j})\}-\frac{\mu}{\Delta t(\Delta x)^2}(U_{i+1,j+1}-2U_{i,j+1}+U_{i-1,j+1}-U_{i+1,j}+2U_{i,j}-U_{i-1,j})=0 \tag{6}$$

for $i=1,2,\ldots,N-1$ and $j=0,1,\ldots,J$ with a truncation error of $O(\Delta t)+O(\Delta x)^2$.