Systems & Control Letters 67 (2014) 40–45

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Linear constrained moving horizon estimator with pre-estimating observer

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ARTICLE INFO

Article history:
Received 6 February 2012
Received in revised form
4 February 2014
Accepted 6 February 2014
Available online 17 March 2014

Keywords:
Moving horizon estimation
State constraints
Input-to-state stability

ABSTRACT

In this paper, a constrained moving horizon estimation (MHE) strategy for linear systems is proposed. Recently, the use of a pre-estimating linear observer in the forward prediction equations in the MHE cost function has been proposed in order to reduce the effects of uncertainty. Here we introduce state constraints within this formulation and investigate stability properties in the presence of bounded disturbances and noise. The robustness and performance of the proposed observer is demonstrated with a simulation example.

1. Introduction

The moving horizon state estimator (MHE) has recently received much attention in the literature, e.g., [1–4]. The idea of MHE is to estimate the current states by solving a least squares optimization problem, which penalizes the deviation between the measurements and predicted outputs, and possibly the distance from the estimated state and an *a priori* state estimate. Since the MHE is based on a window of the most recent data, it can provide a high degree of robustness in the presence of uncertainties such as noise, disturbances and modeling errors; see [3].

Recently, the authors proposed an improved linear MHE approach [5] with a pre-estimating linear estimator instead of open loop forward prediction equations normally being used in the cost functions and constraints of the MHE. This development was made in spirit to the so-called pre-stabilizing model predictive control [6,7], where the control sequence is parameterized as perturbations to a given pre-stabilizing feedback gain. In the MHE, the injection term is parameterized as perturbations to a linear pre-estimating observer. Hence, the states are estimated by a forward simulation with a pre-estimator before optimized in the MHE. Its motivation is to reduce the accumulation of estimation errors on the horizon by using the output injection feedback for stabilization and shaping the dynamics. Moreover, an important effect of the pre-estimation approach compared to, e.g., [1] is that additional variables may not always need to be introduced in the optimization problem in order to account for the unknown disturbances that in our case are implicitly estimated and to some extent accounted for by the pre-estimating linear observer. This greatly reduces the computational complexity of the approach. The use of pre-estimation distinguishes the approach in [5] and the present paper from the approaches known in the literature, e.g., [1–4]. The pre-estimator leads to a different set of tuning parameters that can be used to tune the performance, possibly achieving better performance than other approaches that are parameterized and tuned differently.

In this paper, the method of Sui et al. [5] is extended to take into account state constraints, which can be expected to improve performance as in [1,2,8]. Moreover, instead of using scalar weight parameters, weight matrices are introduced into the cost function. It is shown that the estimated error is input-to-state-stable (ISS) with respect to measurement noise and disturbance inputs.

The outline of the paper is as follows. After Introduction, some preliminaries are given in Section 2. The proposed constrained linear MHE is formulated in Section 3, and its convergence is investigated in Section 4. The robust performance of the proposed observer is demonstrated in a simulation example in Section 5, before some conclusions are given in Section 6.

1.1. Notation and nomenclature

A positive definite (semi-positive definite) square matrix A is denoted by $A > 0$ ($A \geq 0$). $\|x\|_A^2 = x^T A x$ with $A \geq 0$. Let $\rho(A)$

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http://dx.doi.org/10.1016/j.sysconle.2014.02.003
0167-6911/

denote the spectral radius of a square matrix A. For two vectors $x \in R^n$ and $y \in R^m$ we let $\mathrm{col}(x, y)$ denote the column vector in R^{n+m} where x and y are stacked into a single column. A set $X \subset R^n$ is said to be a $\mathscr{C}$ set if it is a compact and convex set that contains the origin in its (non-empty) interior. Suppose $X, Y \subset R^n$; then the P-difference of X and Y is $X \ominus Y = \{z \in R^n : z + y \in X, \forall y \in Y\}$. A function $\varphi : R^+ \to R^+$ is called a K-function if $\varphi(0) = 0$ and it is strictly increasing. A function $\varphi : R^+ \to R^+$ is called a K_∞-function if $\varphi \in K$ and it is radially unbounded. A function $\beta : R^+ \times R^+ \to R^+$ is called a KL-function if for each fixed $k \in R^+$, $\beta(\cdot, k) \in K$ and for each fixed $s \in R^+$, $\beta(s, \cdot)$ is non-increasing and $\lim_{k\to\infty} \beta(s, k) = 0$. For a sequence $\{z_j\}$ for $j \geq 0$, $z_{[t]}$ denotes the truncation of $\{z_j\}$ at time t, i.e. $z_{[t]} = \{z_j\}$ for $0 \leq j \leq t$. A polyhedron is the (convex) intersection of a finite number of open and/or closed half-spaces and a polytope is the closed and bounded polyhedron.

2. Background

Consider the following discrete-time linear time-invariant system

$$x_{t+1} = Ax_t + Bu_t + \xi_t, \tag{1}$$

$$y_t = Cx_t + \eta_t, \tag{2}$$

where $x_t \in X \subseteq R^{n_x}$, $u_t \in R^{n_u}$ and $y_t \in R^{n_y}$ are the state, input and the measurement, respectively, $\xi_t \in R^{n_x}$ is an unknown disturbance, $\eta_t \in R^{n_y}$ is unknown measurement noise, and t is the discrete time index. It is assumed that the disturbance ξ_t and noise η_t lie in the $\mathscr{C}$ sets Ξ and Σ, respectively, and X is a known set which will be used to define state constraints on the state estimation problem. We assume that the input and measurement data are bounded.

While [5] considers detectable systems, we consider without loss of generality observable systems:

(A1) the pair (A, C) is observable.

By decomposing a detectable linear system into observable and unobservable sub-systems, the present design approach is directly extended with an open loop observer for the asymptotically stable unobservable subsystem.

2.1. Linear observer

A linear time-invariant filter (e.g. the Luenberger observer or the stationary Kalman filter) estimates the state according to

$$\hat{x}_{t+1} = A\hat{x}_t + Bu_t + L(y_t - \hat{y}_t), \tag{3a}$$

$$\hat{y}_t = C\hat{x}_t, \tag{3b}$$

where $\hat{x}_t \in R^{n_x}$ is the current observer state, $\hat{y}_t \in R^{n_y}$ is the current observer output estimate, and the observer gain matrix is defined by $L \in R^{n_x \times n_y}$ such that the error dynamics characterized by $\Phi := A - LC$ are asymptotically stable:

(A2) L satisfies $\rho(\Phi) < 1$.

The estimated state $\hat{x}$ satisfies the following uncertain dynamics

$$\hat{x}_{t+1} = A\hat{x}_t + Bu_t + LC\tilde{e}_t + L\eta_t, \tag{4}$$

while the state estimation error $\tilde{e}_t = x_t - \hat{x}_t$ satisfies

$$\tilde{e}_{t+1} = \Phi\tilde{e}_t + d_t, \tag{5}$$

where $d_t = \xi_t - L\eta_t$. Thus there always exists a $\mathscr{C}$ set D such that $d_t \in D$.

Definition 1 (*[9]*). A set $T \subset R^{n_x}$ is disturbance invariant (d-invariant) for the system $x_{t+1} = Ax_t + \xi_t$ and the constraint set (X, Ξ) if $T \subseteq X$ and $x_{t+1} \in T$ for all $\xi_t \in \Xi$ and $x_t \in T$.

Due to $\rho(\Phi) < 1$, there exists a set E such that it is d-invariant for the system (5). It implies that if $\tilde{e}_0 \in E$, then $\tilde{e}_t \in E$, $\forall t \geq 0$. In this paper the set E is chosen as the outer bound of the minimal d-invariant set of system (5). Methods to compute such a set for linear systems have appeared in the literature; see for example, [10,11].

2.2. Input-to-state stability

The concept of input-to-state stability (ISS) has been widely used in stability analysis and control synthesis. Recently, some of the well-established results in ISS for continuous time nonlinear system have been extended to discrete-time nonlinear systems [12]. Some of the results of ISS are reviewed in this section.

Consider the following discrete-time nonlinear system:

$$z_{t+1} = g(z_t, u_t) \tag{6}$$

or $z^+ = g(z, u)$, where $z_t \in Z \subseteq R^{n_z}$ and $u_t \in U$. We define the notion of input-to-state stability [12].

Definition 2. The system (6) is ISS; if there exists a KL-function $\tilde{\theta}$, and a K_∞-function γ_u such that for any $t \geq 0$, any initial conditions $z_0 \in X$ and any $u_{[t-1]}$ with $u_j \in U$, $0 \leq j \leq t-1$, then the following is true:

$$\|z_t\| \leq \tilde{\theta}(\|z_0\|, t) + \gamma_u(\|u_{[t-1]}\|). \tag{7}$$

Definition 3. A continuous function $V : R^{n_z} \to R \geq 0$ is called an *ISS*-Lyapunov function for the system (6) if the following holds:

1. $V(0) = 0$.
2. There exist K_∞-functions α_1, α_2 such that for any z,

$$\alpha_1(\|z\|) \leq V(z) \leq \alpha_2(\|z\|). \tag{8}$$

3. There exists a K-function σ, such that for any z and any input signals u

$$V(z^+) - V(z) \leq -\alpha_3(\|z\|) + \sigma(\|u\|) \tag{9}$$

or

$$V(z^+) - V(z) \leq -\alpha_3(\|z^+\|) + \sigma(\|u\|) \tag{10}$$

with α_3 positive on R^+.

Theorem 1 (*[12]*). *If the system (6) admits an ISS-Lyapunov function, then it is ISS.*

Note that an ISS system is globally asymptotically stable in the absence of input or if the input is decaying. If the input is merely bounded then the evolution of the system is ultimately bounded in a set whose size depends on the bound of the input.

3. Linear moving horizon estimation

An MHE recursively estimates the state based on a finite window of current and past data. It estimates the state vectors $x_{t-N}, \ldots, x_t$ at any time $t = N, N+1, \ldots,$ on the basis of the a priori estimate $\bar{x}_{t-N,t}$ and the current information vector defined as

$$I_t = \mathrm{col}(y_{t-N}, \ldots, y_t, u_{t-N}, \ldots, u_{t-1}),$$

where $N + 1$ is the window length or horizon.

Like in [5], we formulate the MHE strategy by introducing a linear observer as a pre-estimating observer, since the injection term will reduce the effect of uncertainty (e.g. model errors, disturbances and noise) in the a priori estimate and predictions, and thereby contribute to improve the accuracy. The proposed