Systems & Control Letters 67 (2014) 65–70

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

On infinite-horizon sensor scheduling

Y. Mo [a,*], E. Garone [b], B. Sinopoli [c]

[a] Department of Control and Dynamical Systems, California Institute of Technology, 1200 E. California Blvd, Pasadena, CA, United States
[b] Service d'Automatique et d'Analyse des Systèmes, Universitè Libre de Bruxelles, 50 Av. F.D. Roosevelt, B-1050 Brussels, Belgium
[c] Department of Electrical and Computer Engineering, Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA, United States

ARTICLE INFO

Article history:
Received 10 February 2013
Received in revised form
15 January 2014
Accepted 4 February 2014
Available online 17 March 2014

Keywords:
Wireless sensor networks
Kalman filtering
Riccati equation

ABSTRACT

In this paper we consider the problem of infinite-horizon sensor scheduling for estimation in linear Gaussian systems. Due to possible channel capacity, energy budget or topological constraints, it is assumed that at each time step only a subset of the available sensors can be selected to send their observations to the fusion center, where the state of the system is estimated by means of a Kalman filter. Several important properties of the infinite-horizon schedules will be presented in this paper. In particular, we prove that the infinite-horizon average estimation error and the boundedness of a schedule are independent of the initial covariance matrix. We further provide a constructive proof that any feasible schedule with finite average estimation error can be arbitrarily approximated by a bounded periodic schedule. We later generalized our result to lossy networks. These theoretical results provide valuable insights and guidelines for the design of computationally efficient sensor scheduling policies.

1. Introduction

Sensor networks span a wide spectrum of applications, e.g., environment and habitat monitoring, health care, home and office automation, and traffic control [1]. In many of these applications, a centralized fusion center is implemented to collect and process the measurements for estimation purposes. Sensor nodes are typically battery powered, and therefore energy constrained. Furthermore, their radios are low-power and may be subject to interference and fading. As a result of the bandwidth and energy constraints, it is not advisable, and sometimes infeasible, for all the sensors to communicate with the fusion center within each sampling period. Thus, it is of significant interest to determine sensor scheduling policies able to tradeoff energy/bandwidth consumption and estimation quality.

Sensor network energy consumption minimization and lifetime maximization problems have been active areas of research in recent years. Sensor networks' energy management is typically carried out via efficient MAC protocols [2] or via efficient scheduling of sensor states [3,4]. Xue and Ganz [5] showed that the lifetime of sensor networks is influenced by transmission schemes, network density and transceiver parameters with different constraints on network mobility, position awareness and maximum transmission ranges. Chamam and Pierre [6] proposed a sensor scheduling scheme capable of optimally putting sensors in active or inactive modes. Shi et al. [7] considered sensor energy minimization as a mean to maximize the network lifetime while guaranteeing a desired quality of the estimation accuracy. The same authors further proposed a sensor tree scheduling algorithm [8] which leads to longer network lifetimes.

Performance optimization for sensor networks under given energy constraints, which can be seen as the dual problem of network energy minimization, has also been studied by several researchers. Such constrained optimization problem has been studied for continuous-time linear systems by Miller and Runggaldier [9] and Mehra [10]. Krishnamurthy [11] derived the optimal sensor scheduling for the estimation of a Hidden Markov Model based system. For discrete-time linear systems, approaches using dynamic programming [12], greedy algorithms [13], convex optimization [14–16] and branch and bound [17] have been proposed to find the optimal or suboptimal sensor scheduling over finite time horizons. In general, the sensor scheduling problem is a combinatorial optimization problem [18] and thus the exact optimal solution over long time horizons is computationally intractable. However, the exact optimal schedule can be computed in some very particular cases. For instance, Shi and Zhang [19] and Hovareshti et al. [20] prove that under certain conditions, the optimal

* Corresponding author. Tel.: +1 4122685231.
E-mail addresses: yilinmo@caltech.edu (Y. Mo), egarone@ulb.ac.be (E. Garone), brunos@ece.cmu.edu (B. Sinopoli).

http://dx.doi.org/10.1016/j.sysconle.2014.02.002
0167-6911/

Table 1
Notations.

S	Set of sensors
$\mathcal{S}$	Collection of all eligible subsets of S
$\mathcal{I}_k$	Subset of sensors selected at time k
σ	An infinite sensor schedule in the form of $(\mathcal{I}_1, \mathcal{I}_2, \ldots)$
Σ	The covariance of the initial state x_0
Q	The covariance of process noise
R	The covariance of measurement noise
$J(\sigma, \Sigma)$	The average trace of the error covariance matrix
$r_i(\sigma)$	The average communication rate of sensor i

infinite-horizon schedule is periodic for a system with two smart sensors.

Power control has also been studied [21,22] to increase the energy efficiency of sensors. To this end, a sensor could use a lower power level to communicate information, which results in either a lower SNR, an increase in communication delay or a larger packet drop probability. Conceptually, for sensors with finitely many power levels, a virtual sensor could be assigned to each power level. Hence, the usage of power control can be seen as a special case of sensor scheduling.

In most of the works cited above, the optimal schedule can only be computed for linear systems over a finite-horizon, while only for specific systems an infinite-horizon policy can be derived. Moreover, for general systems, the solution is usually given as the result of an optimization problem and thus implicit. In this paper, we consider the problem of sensor scheduling for state estimation of linear LTI systems with Gaussian noise over an infinite-horizon. In particular we focus on proving several fundamental properties that can be used as guidelines for the analysis and design of infinite-horizon sensor schedules. In particular, we prove the following two propositions concerning scheduling policies:

1. The average estimation error of a schedule is independent of the initial covariance of x_0.
2. Any schedule that has a bounded average estimation error can be arbitrarily approximated (both in terms of average estimation error and communication rate) by bounded periodic schedules.

These results have important practical consequences as bounded periodic schedules are easier to compute than general ones.

The rest of the paper is organized as follows: in Section 2, we formulate the infinite-horizon sensor scheduling problem. In Section 3, we prove that the average estimation covariance is independent of the initial conditions. We further provide a constructive proof that any feasible schedule can be arbitrarily approximated by bounded periodic schedules in Section 4. We then generalize our results to lossy networks in Section 5. A numerical example is presented in Section 6 to illustrate the performance of periodic schedules. Finally, Section 7 concludes the paper.
Notations: We summarize the notations used in this paper in Table 1.

2. Problem formulation

Consider the following discrete-time LTI system

$$x_{k+1} = Ax_k + w_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$ represents the state and $w_k \in \mathbb{R}^n$ the process noise. It is assumed that w_k and x_0 are independent Gaussian random vectors, $x_0 \sim \mathcal{N}(0, \ \Sigma)$ and $w_k \sim \mathcal{N}(0, \ Q)$, where $\Sigma, \ Q > 0$.[1] A wireless sensor network composed of m sensing devices $S = \{s_1, \ldots, s_m\}$ and one fusion center is used to monitor the state of system (1). The measurement equation is

$$y_k = Cx_k + v_k, \tag{2}$$

where $y_k = [y_{k,1}, y_{k,2}, \ldots, y_{k,m}]' \in \mathbb{R}^m$ is the measurement vector.[2] Each element $y_{k,i}$ represents the measurement of sensor i at time k, $C = \left[C_1', \ldots, C_m'\right]'$ is the observation matrix and the matrix pair (C, A) is assumed observable, $v_k \sim \mathcal{N}(0, \ R)$ is the measurement noise, assumed to be independent of x_0 and w_k.

Suppose that due to energy, bandwidth or topological constraints, only a subset of sensors can be chosen to send their measurements to the fusion center. Denote the collection of all eligible subsets as $\mathcal{S} \subseteq \mathcal{P}(S)$, where $\mathcal{P}(S)$ denotes the power set of S, i.e., the collection of all subsets of S.

For any $\mathcal{I} = \{s_{i_1}, \ldots, s_{i_l}\} \in \mathcal{S}$, we define the selection matrix $\Gamma(\mathcal{I})$

$$\Gamma(\mathcal{I}) \triangleq \left[e_{i_1}, \ldots, e_{i_l}\right]',$$

where e_i is the ith vector of the canonical basis, i.e. a vector with entries 0 everywhere, except a 1 at the ith entry. By means of this selection matrix we can define the matrices

$$C(\mathcal{I}) \triangleq \Gamma(\mathcal{I})C, \qquad R(\mathcal{I}) \triangleq \Gamma(\mathcal{I})R\Gamma(\mathcal{I})',$$

that allow one to define the matrix-valued function $g(X, \mathcal{I})$ as

$$g(X, \mathcal{I}) \triangleq \left[(AXA' + Q)^{-1} + C(\mathcal{I})'R(\mathcal{I})^{-1}C(\mathcal{I})\right]^{-1}.$$

A schedule is defined as an infinite sequence of $\sigma \triangleq (\mathcal{I}_1, \mathcal{I}_2, \ldots)$ satisfying the constraint $\mathcal{I}_k \in \mathcal{S}$. Clearly, if a schedule σ is used, the covariance of the Kalman filter satisfies the following equation:

$$P_k = g(P_{k-1}, \mathcal{I}_k), \qquad P_0 = \Sigma. \tag{3}$$

Remark 1. In case of quantized measurements, You et al. [23] and Msechu et al. [24] propose a Quantized Kalman Filtering (QKF) algorithm, where the approximated P_k follows a modified Riccati equation similar to (3). As a result, all the results discussed in this paper can be generalized to QKF.

Since P_k is a function of both the sensor schedule σ and the initial condition Σ, we will denote P_k as $P_k(\sigma, \Sigma)$. Let us define the cost function $J(\sigma, \Sigma)$ as

$$J(\sigma, \Sigma) \triangleq \limsup_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \mathrm{tr}(P_k(\sigma, \Sigma)).$$

$J(\sigma, \Sigma)$ can be seen as the average estimation error. Moreover, let us define the average communication rate of sensor i as

$$r_i(\sigma) \triangleq \limsup_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \mathbb{I}_{s_i \in \mathcal{I}_k},$$

where $\mathbb{I}$ is the indicator function.

Remark 2. Our formulation can address a large class of sensor selection problems. In particular, the set $\mathcal{S}$ can be used to characterize the topological and communication constraints of the network, while variables r_i define the average usage of the sensors, which can be used to define energy constraints on sensors.

We have the following definitions:

Definition 1. A schedule σ is called *feasible* if for all initial conditions $\Sigma, J(\sigma, \Sigma) < \infty$.

Definition 2. A schedule σ is called *bounded* if for all initial conditions $\Sigma > 0$, there exists a matrix $M(\Sigma)$, such that $P_k(\sigma, \Sigma) \leq M(\Sigma)$ for all k.[3]

[1] All the comparisons between matrices are in the sense of positive semidefinite.

[2] The $'$ on a matrix always means transpose.

[3] Note that while boundedness clearly implies feasibility, the converse is not always true. It is enough to consider the sequence of P_k is $\{1, 0, 2, 0, 0,$