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Peristaltic motion of a Jeffrey fluid under the effect of a magnetic field in a tube

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Abstract

This study is concerned with the analysis of peristaltic motion of a Jeffrey fluid in a tube with sinusoidal wave travelling down its wall. The fluid is electrically conducting in the presence of a uniform magnetic field. Analytic solution is carried out for long wavelength and low Reynolds number considerations. The expressions for stream function, axial velocity and axial pressure gradient have been obtained. The results for pressure rise and frictional force per wavelength obtained in the analysis have been evaluated numerically and discussed briefly. The significance of the present model over the existing models has been pointed out by comparing the results with other theories. It is further noted that under the long wavelength approximation, the retardation time has no effect in the present analysis.

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Keywords: Peristalsis; Magnetohydrodynamic fluid; Jeffrey fluid

1. Introduction

The dynamics of the fluid transport by peristaltic motion of the confining walls has received a careful study in the literature. The need for peristaltic pumping may arise in circumstances where it is desirable to avoid using any internal moving parts such as pistons in a pumping process. The peristalsis is also well known to the physiologists to be one of the major mechanisms of fluid transport in a biological system and appears in urine transport from kidney to bladder through the ureter, movement of chyme in the gastrointestinal tract, the movement of spermatozoa in the ductus efferentes of the male reproductive tract and the ovum in the female fallopian tube, the locomotion of some worms, transport of lymph in the lymphatic vessels and vasomotion of small blood vessels such as arterioles, venules and capillaries. Technical roller and finger pumps also operate according to this rule.

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The behavior of most of the physiological fluids is known to be non-Newtonian. Several models have been proposed to explain the non-Newtonian behavior of fluids. Various attempts [1–8] are made to solve the extremely complex equations of motion of non-Newtonian fluids. The good number of recent investigations [9–18] on the peristalsis of non-Newtonian fluids have been presented with various perspectives, in channels or tubes. Most of the analytic studies are asymptotic expansions with small Reynolds number, wave number, amplitude ratio as a perturbation parameter. Siddiqui et al. [19] examined the peristaltic motion of a magnetohydrodynamic Newtonian fluid in a tube by taking long wavelength approximation. More recently Hayat and Ali [20] studied the peristaltic motion of a third order fluid in a tube under long wavelength and small Deborah number approximation.

However, no attempt has been made to discuss the peristaltic motion of a magnetohydrodynamic (MHD) non-Newtonian fluid in a tube which holds for all values of non-Newtonian parameters. In the present analysis, such an attempt has been made. The liquid considered is of the Jeffrey type and is electrically conducting. This shows worthwhile being the first attempt for MHD non-Newtonian flow in a tube for all values of the rheological parameters. The Jeffrey model is relatively simpler linear model using time derivatives instead of convected derivatives for example the Oldroyd-B model does, it represents a rheology different from the Newtonian. Although more sophisticated viscoelastic models than the Jeffrey model exist, in a first study of the MHD peristaltic motion of a non-Newtonian fluid in circular cylindrical tube, the choice of Jeffrey fluid model is motivated by the following.

In spite of its relative simplicity, the Jeffrey model can indicate the changes of the rheology on the peristaltic flow even under the assumption of large wavelength, low Reynolds number and small or large amplitude ratio. In Newtonian fluid, Mekheimer [18] studied the MHD peristaltic flow in a channel under the assumption of small wave number. Therefore, at least in an initial study, this motivate an analytic study of MHD peristaltic non-Newtonian tube flow that holds for all non-Newtonian parameters. By choosing the Jeffrey fluid model it become possible to treat both the MHD Newtonian and non-Newtonian problems analytically under long wavelength and low Reynolds number consideration. Considering the blood as a MHD fluid, it may be possible to control blood pressure and its flow behavior by using an appropriate magnetic field. The influence of magnetic field may also be utilized as a blood pump for cardiac operations for blood flow in arterial stenosis or arteriosclerosis.

This work is arranged as follows. In Section 2, the problem of interest is formulated and governing equations are modeled. Section 3 deals with the solution of the problem under long wavelength and low Reynolds number assumptions. Section 4 synthesis the results obtained. Concluding remarks are presented in Section 5.

2. Mathematical modeling

Consider the axisymmetric flow of a Jeffrey fluid in a uniform circular tube with a sinusoidal peristaltic wave of small amplitude travelling down its wall (see Fig. 1). The geometry of wall surface is therefore described as

$$\bar{h}(\overline{Z},\bar{t}) = a + b\sin\left[\frac{2\pi}{\lambda}(\overline{Z}-c\bar{t})\right] \tag{1}$$

in which a is the average radius of the undisturbed tube, b is the amplitude of the peristaltic wave, λ is the wavelength, c is the wave propagation speed, and $\bar{t}$ is the time. $\overline{R}$ and $\overline{Z}$ are the cylindrical coordinate with $\overline{Z}$ measured along the axis of the tube and $\overline{R}$ is in the radial direction. Let $(\overline{U},\overline{W})$ be the velocity components in fixed frame of reference $(\overline{R},\overline{Z})$. We further assume that wall is extensible and fluid is electrically conducting. A uniform magnetic field $\overline{\mathbf{B}}_0$ is applied in the transverse direction to the flow. The magnetic Reynolds number is taken small so that induced magnetic field is neglected. The constitutive equations for an incompressible Jeffrey fluid are

$$\overline{\mathbf{T}} = -\bar{p}\overline{\mathbf{I}} + \overline{\mathbf{S}}, \tag{2}$$

$$\overline{\mathbf{S}} = \frac{\mu}{1+\lambda_1}(\dot{\bar{\gamma}} + \lambda_2\ddot{\bar{\gamma}}), \tag{3}$$

where $\overline{\mathbf{T}}$ and $\overline{\mathbf{S}}$ are Cauchy stress tensor and extra stress tensor respectively, $\bar{p}$ is the pressure, $\overline{\mathbf{I}}$ is the identity tensor, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, μ is the dynamic viscosity, $\dot{\gamma}$ is the shear rate and dots over the quantities indicate differentiation with respect to time.