Systems & Control Letters 64 (2014) 79–85

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Data-driven asymptotic stabilization for discrete-time nonlinear systems[☆]

Yongqiang Li, Zhongsheng Hou [*]

Advanced Control Systems Lab, School of Electronic and Information Engineering, Beijing Jiaotong University, Beijing 100044, China

ARTICLE INFO

Article history:
Received 30 August 2012
Received in revised form
30 October 2013
Accepted 5 November 2013
Available online 31 December 2013

Keywords:
Control Lyapunov function
Domain of attraction
Data-driven control
Stabilization
Nonlinear
Discrete-time

ABSTRACT

In this paper, we propose a data-driven feedback controller design method based on Lyapunov approach, which can guarantee the asymptotic stability of the closed-loop and enlarge the estimate of domain of attraction (DOA) for the closed-loop. First, sufficient conditions for a feedback controller asymptotically stabilizing the discrete-time nonlinear plant are proposed. That is, if a feedback controller belongs to an open set consisting of pairs of control input and state, whose elements can make the difference of a control Lyapunov function (CLF) to be negative-definite, then the controller asymptotically stabilizes the plant. Then, for a given CLF candidate, an algorithm, to estimate the open set only using data, is proposed. With the estimate, it is checked whether the candidate is or is not a CLF. If it is, a feedback controller is designed just using data, which satisfies sufficient conditions mentioned above. Finally, the estimate of DOA for closed-loop is enlarged by finding an appropriate CLF from a CLF candidate set based on data. Because the controller is designed directly from data, complexity in building the model and modeling error are avoided.

1. Introduction

Lyapunov approach provides a powerful framework for analyzing the stability of nonlinear dynamical systems as well as designing feedback controllers that guarantee closed-loop system stability [1,2]. The synthesis typically relies on the co-design of a CLF and a state feedback controller. Historically, this is an important but challenging problem for the general class of nonlinear systems [3]. The main bottleneck to the success of these methods lies in the construction of CLF. Moreover, the DOA, an invariant set characterizing stabilizable area around the equilibrium, needs further investigation because global stabilization is difficult to achieve in practical applications [4]. There have been a number of studies to solve this problem. These results can be divided into two categories: model-based and data-driven.

Among model-based approaches, one simple approach to obtain a quadratic CLF is solving the Riccati equation associated with the linearized system of the nonlinear system, which often leads to a small DOA for the closed-loop due to approximation errors. Based on sum-of-squares (SOS) programming [5,6], a polynomial CLF can be constructed and an enlarged estimate of DOA is simultaneously obtained [7–9]. However, SOS based methods can handle polynomial or rational systems only. SOS programming is extended to non-polynomial systems by variable transformation with algebraic constraints [10]. In [4], a fractional programming problem is formulated to construct a CLF for non-polynomial systems. The main disadvantage of model-based approaches is that the model of the nonlinear plant is the prerequisite and the model must be control-affine.

In fact, there are lots of plants which are hard to be effectively modeled. So designing controllers directly from data bypassing the modeling step, called Data-driven Control Approach, is promising and has received much attention recently. Many data-driven control approaches could be found, such as unfalsified control (UC) [11], model-free adaptive control (MFAC) [12,13], *etc*. The main ideas of these approaches are quite different, but they do not require a plant model and do directly use data.

Among data-driven approaches, the control Lyapunov measure approach [14] deals with a similar problem with the one in this paper. Instead of a point-wise notion of the CLF, a control Lyapunov measure is constructed in the term of measure-theory for a nonlinear plant. This approach is data-driven, even though the authors do not say so. In this approach, the model is used to generate data for computing the Markov matrix. The disadvantage of this approach is that it leads to weaker coarse stability while our conclusion on stability is exact.

In this paper, we propose a data-driven asymptotic stabilization for discrete-time nonlinear systems, where a feedback controller asymptotically stabilizing the plant is obtained directly from data and the DOA of the closed-loop is enlarged. First, sufficient conditions for the feedback controller asymptotically stabilizing

☆ This research was supported by the State Key Program (No. 60834001) and the Major Program of International Cooperation and Exchanges (No. 61120106009) of National Natural Science Foundation of China.

* Corresponding author. Tel.: +86 10 51688617.
E-mail addresses: lyqpluto@gmail.com (Y. Li), zhshhou@bjtu.edu.cn (Z. Hou).

0167-6911/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.sysconle.2013.11.003

the plant are proposed. For a given CLF of the plant, if a feedback controller belongs to an open set consisting of pairs of control input and state of the plant, whose elements make the difference of the CLF to be negative-definite, the feedback controller can asymptotically stabilize the plant. However, under traditional controller design frameworks, it is hard to obtain the set for general nonlinear plants. Then, based on a data set collected from the plant and a given CLF candidate, an estimate of the set can be obtained. The idea of estimating the set, similar to set oriented numerical methods [15], is covering the set by a finite number of cells which contains data points satisfying some specific conditions. From the estimate of the set, it is easy to check whether the candidate is or is not a CLF. If it is, a feedback controller is designed using data, which satisfies sufficient conditions mentioned above. Finally, the estimate of DOA for closed-loop is enlarged by finding an appropriate CLF from a CLF candidate set based on data. An unconstrained nonlinear optimization problem, which can be solved by metaheuristic optimizers, is proposed to find the appropriate CLF. In our method, we directly use data bypassing the modeling step. Hence, complexity in building the model and modeling error are avoided.

This paper is organized as follows. In Section 2 the control problem is formulated. In Section 3, sufficient conditions for asymptotic stabilization and estimation of DOA for the closed-loop are introduced. In Section 4, the data-driven asymptotic stabilization is derived. In Section 5, the estimate of DOA for the closed-loop is enlarged by selecting an appropriate CLF from a CLF candidate set. Finally, in Section 6, the conclusion is drawn and further works are summarized.

Notation: $\mathbb{R}$ represents the set of real numbers. $\mathbb{R}_+$ represents the set of positive real numbers. $\overline{\mathbb{R}}_+$ represents $\mathbb{R}_+ \cup \{0\}$. $\mathbb{Z}_+$ represents the set of positive integer numbers. $\overline{\mathbb{Z}}_+$ represents $\mathbb{Z}_+ \cup \{0\}$. $\mathbb{R}^n$ represents the set of real vectors with n elements. For a vector $x \in \mathbb{R}^n$, $\|x\|$ represents $\sqrt{x^T x}$. For a vector $x \in \mathbb{R}^n$, $x_{(i)}$ represents the i-th element of x, $i = 1, 2, \ldots, n$. For a domain $\mathcal{X} \subset \mathbb{R}^n$, $m(\mathcal{X})$ represents the Lebesgue measure of $\mathcal{X}$ (in Euclidean space, it is the volume of $\mathcal{X}$).

2. Problem formulation

Consider the nonlinear discrete-time system

$$x(k+1) = f\left(x(k), u(k)\right), \qquad x(0) = x_0, \quad k \in \overline{\mathbb{Z}}_+, \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the control input, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is an unknown piecewise continuous function satisfying $0 = f(0, 0)$ and is asymptotically stabilizable at the origin.

Although f is unknown, we have a data set

$$\mathcal{T} = \left\{T_i, i = 1, 2, \ldots, N\right\} \tag{2}$$

collected from the plant (1) without measurement noises, where $T_i = \{x_i(0 : K_i), u_i(0 : K_i - 1)\}$ consists of the state trajectory and the control sequence, $x_i(0 : K_i) = (x_i(0), \ldots, x_i(K_i))$ is the state trajectory of the plant (1) with the control sequence $u_i(0 : K_i - 1) = (u_i(0), \ldots, u_i(K_i - 1))$, $K_i \in \mathbb{Z}_+$ is the length of the state trajectory and $N \in \mathbb{Z}_+$ is the number of state trajectories in $\mathcal{T}$.

Remark 1. In practice, collecting a data set, which contains enough dynamic information of the nonlinear plant, is not an easy work. As mentioned in [16], the data set containing enough information should have two characteristics: sufficiency and completeness. The sufficiency means that sample points in the data set evenly spread over the range of interests in the sample space, and the completeness means that sample points are dense almost everywhere. However, quantitative analysis, for whether a data set is sufficient and complete, is an open problem. In this paper, we do not deal with this problem. In order to warrant the acquisition of enough information of plant (1), we give the following assumption. □

Assumption 1. The number of state trajectories N is sufficient large and both the initial state $x_i(0) \in \mathbb{R}^n$ and the control input $u_i(k) \in \mathbb{R}^m$ for each trajectory are set to be random values with the uniform distribution on interested ranges when collecting trajectories.

Using the data set $\mathcal{T}$, our control objective is to find a nonlinear feedback controller $u(k) = \mu(x(k))$, where $\mu : \mathbb{R}^n \to \mathbb{R}^m$ is a function, such that the closed-loop $x(k+1) = f(x(k), \mu(x(k)))$ is asymptotically stable at the origin and the estimate of DOA for the closed-loop is as large as possible.

3. Sufficient conditions for asymptotic stabilization and estimation of DOA for closed-loops

In this section, first, we introduce sufficient conditions for estimation of DOA for nonlinear discrete-time systems without control input in Lemma 1. Since the theory for nonlinear discrete-time systems closely parallels the theory for nonlinear continuous-time systems, many of the results are similar [1]. However, for the estimate of DOA by Lyapunov function, the discrete-time result deviates markedly from its continuous-time counterpart as illuminating in Remark 2. Then, sufficient conditions for asymptotic stabilization and estimation of DOA for closed-loops are introduced in Lemma 2, which is a theoretical cornerstone of our method.

3.1. Sufficient conditions for estimation of DOA for systems without control input

Consider the following nonlinear discrete-time system without control input

$$x(k+1) = f_a\left(x(k)\right), \qquad x(0) = x_0, \quad k \in \overline{\mathbb{Z}}_+, \tag{3}$$

where $f_a : \mathbb{R}^n \to \mathbb{R}^n$ is a piecewise continuous function and $f_a(0) = 0$. Let $\phi(x_0, k)$ denote the solution to (3) at time k with the initial condition $\phi(x_0, 0) = x_0$. If the origin is asymptotically stable but not globally attractive, one often wants to know how far from the origin the state trajectory can still converge to the origin as $k \to \infty$. This gives rise to the conception of domain of attraction. The DOA $\mathcal{X}_{doa}$ of the origin for (3) is

$$\mathcal{X}_{doa} = \left\{x_0 \in \mathbb{R}^n \mid \lim_{k\to\infty} \phi(x_0, k) = 0\right\}.$$

Even though the function f_a is known, computing the exact DOA might be difficult or even impossible. Via much simpler procedures, we can find an estimates of DOA by using a Lyapunov function. We propose the following lemma, which gives an estimate of DOA and is the discrete-time version of Lemma 1 in [6].

Lemma 1. *For system (3), assume that there exists a continuous positive-definite function $V : \mathbb{R}^n \to \mathbb{R}$ and a constant $\gamma \in \overline{\mathbb{R}}_+$ such that*

$$\mathcal{X}_{V,\gamma} = \left\{x \in \mathbb{R}^n \mid V(x) \le \gamma\right\} \quad \textit{is bounded}, \tag{4}$$

$$\mathcal{X}_{V,\gamma} \setminus \left\{0\right\} \subset \mathcal{X}_{V,f_a} = \left\{x \in \mathbb{R}^n \mid V\left(f_a(x)\right) - V(x) < 0\right\}, \tag{5}$$

then $\forall x_0 \in \mathcal{X}_{V,\gamma}$, the solution $\phi(x_0, k)$ of (3) satisfies, $\forall k \in \overline{\mathbb{Z}}_+$, $\phi(x_0, k) \in \mathcal{X}_{V,\gamma}$ and $\lim_{k\to\infty} \phi(x_0, k) = 0$.

Lemma 1 shows that the γ-level set $\mathcal{X}_{V,\gamma}$ of the Lyapunov function V is an invariant subset of DOA for the equilibrium point $x = 0$. The proof of Lemma 1 is similar with its continuous-time version and is omitted here.