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Distributed adaptive consensus for multiple mechanical systems with switching topologies and time-varying delay

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ABSTRACT

This paper studies the adaptive consensus problem of networked mechanical systems with time-varying delay and jointly-connected topologies. Two different consensus protocols are proposed. First, we present an adaptive consensus protocol for the connected switching topologies. Based on graph theory, Lyapunov stability theory and switching control theory, the stability of the proposed algorithm is demonstrated. Then we investigate the problem under the more general jointly-connected topologies, and with concurrent time-varying communication delay. The proposed consensus protocol consists of two parts: one is for the connected agents which contains the current states disagreement among them and the other is designed for the isolated agents which contains the states difference between the current and past. A distinctive feature of this work is to address the consensus control problem of mechanical systems with unknown parameters, time-varying delay and switching topologies in a unified theoretical framework. Numerical simulation is provided to demonstrate the effectiveness of the obtained results.

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1. Introduction

Recently, the consensus problem of multi-agent systems (MAS) has attracted considerable attention due to its broad applications in formation control of satellite clusters, cooperative control of unmanned aerial vehicles, distributed optimization of multiple robotic systems, etc. [1–4]. Broadly speaking, consensus means that networked agents reach an agreement regarding certain quantity of interests, which might be the attitude in multi-spacecraft alignment, the heading angle of flocking, or the average in distributed computation.

Two topics on the consensus problem have been extensively studied. One is to address the effects of time-delay which is usually inevitable within communication networks. Results on time delays of MAS include [2,5–7], to name just a few. Most of the existing results are under the assumption of constant communication delays. Unfortunately, in practical implementations, these assumptions are not always satisfied. In fact, communication over networks imposes restrictive constraints that include time-varying, unknown and possibly discontinuous communication delays. The other topic is on the effects of the switching communication topology which is actually very commonplace in practical applications due to communication failures [8,9]. Indeed, the communication topology plays an essential role on the stability of the networked multiple systems, especially when it is disconnected at some time instants. The aforementioned results mainly concentrate on dealing with one of these two topics separately. However, in practical applications, these two factors may coexist, thus it is worthwhile to study the two problems together. Although there have been some results on this issue (see [10–14] for example), most of those studies focus on the linear first-order or second-order integrator multi-agent systems.

On the other hand, the past few years have witnessed a burgeoning interest on synchronization (or coordination) control of nonlinear systems, especially the multiple mechanical systems whose dynamics can be modeled by Euler–Lagrange equations (see [15–26], etc.). For example, [17,18] have studied the dynamic tracking of networked Euler–Lagrange systems with only partial agents communicating with the leader, yet they are all based upon the assumption that there is no time delay within the communication topology. The time delay effects are studied in [19–22] from the perspectives of the passivity property of Euler–Lagrange systems, where the communication topologies are required to be fixed. In addition, the time delay effects are also studied using the Lyapunov theory in [23,24] and the small gain theory in [25], yet the communication topology is still restricted to be fixed. Indeed, due to the inherent nonlinearity of the Euler–Lagrange system, the consensus problem is more challenging for the switching topology compared with its fixed topology counterpart. Therefore, the study on switching topologies of multiple mechanical systems is rare with an exception of [26,27]. In [26], the results are under the

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assumption that the switching topologies are balanced and connected, while in [27], the consensus problem is studied under connected switching network but time delay is not taken into consideration. Furthermore, neither [27] nor [26] considers the jointly-connected topologies, let alone time-varying delay together with the jointly-connected topologies.

Motivated by this, in this paper, we study the consensus algorithms of networked mechanical systems with time-varying delay and switching topologies. In particular, we concentrate on mechanical systems whose dynamics are modeled by Euler–Lagrange equations. Since jointly-connected topologies have disconnected nodes, it is more challenging to study consensus problems on jointly-connected topologies than on connected topologies, especially when time delays are involved. A distinctive feature of this paper is that it comprehensively studies the distributed consensus of Euler–Lagrange systems with coupling time-varying delay, jointly-connected topologies and unknown parameters at the same time. To the best of our knowledge, there is still no result which comprehensively considers these factors together.

This paper is organized as follows. In Section 2, we present the problem formulation and background. The consensus problem of multi-agent mechanical systems with unknown parameters and connected switching topologies is studied in Section 3. In Section 4, we further develop an adaptive consensus protocol which allows for time-varying delay, jointly-connected switching topologies and parametric uncertainties. Simulation results are given in Section 5. Finally, we draw our conclusions in Section 6.

Notation. $\mathbb{R} := (-\infty, \infty)$, $\lambda_m\{A\}$ and $\lambda_M\{A\}$ represent the minimum and maximum eigenvalues of matrix A, respectively. $\|A\|$ is the norm of the matrix A. $|x|$ stands for the Euclidean norm for the of vector $x \in \mathbb{R}^n$. For any function $f : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathbb{L}_\infty$-norm is defined as $\|f\|_\infty = \sup_{t\geq 0} |f(t)|$, and the $\mathbb{L}_2$-norm as $\|f\|_2^2 = \int_0^\infty |f(t)|^2 \, dt$. The $\mathbb{L}_\infty$ and $\mathbb{L}_2$ spaces are defined as the sets $\{f : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|f\|_\infty < \infty\}$ and $\{f : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|f\|_2 < \infty\}$, respectively.

2. Problem formulation and background

2.1. System dynamics

We consider a team of n networked mechanical systems (henceforth called agents) indexed by the set $\mathcal{I} = \{1, \ldots, n\}$. The ith system is described by Euler–Lagrange equation

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i(t) \tag{1}$$

where $q_i \in \mathbb{R}^m$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{m\times m}$ is the symmetric positive-definite inertial matrix, $C_i(q_i, \dot{q}_i)q_i \in \mathbb{R}^m$ is the vector of Coriolis and centrifugal torques, $G_i(q_i)$ is the gravitational torques and $\tau_i(t)$ is the vector of torques produced by the actuators associated with the ith system. Before proceeding, we give some fundamental properties for system (1) that will be extensively exploited in the following [28].

Property 1. *The matrix $M_i(q_i)$ is positive and there exist positive constants λ_m and λ_M such that*

$$\lambda_m I \leq M_i(q_i) \leq \lambda_M I. \tag{2}$$

Property 2. *For any differentiable vector $\zeta_i \in \mathbb{R}^m$, the Lagrangian dynamics are linearly parameterizable which gives that*

$$M_i(q_i)\dot{\zeta}_i + C_i(q_i, \dot{q}_i)\zeta_i + G_i(q_i) = Y_i(q_i, \dot{q}_i, \zeta_i, \dot{\zeta}_i)\Theta_i \tag{3}$$

where $\Theta_i \in \mathbb{R}^k$ is a constant vector of parameters whose elements include the link masses, moments of inertial, etc., and $Y_i(\cdot) \in \mathbb{R}^{m\times k}$ is the matrix of known functions of the generalized coordinates and their higher derivatives.

Property 3. *Under an appropriate definition of the matrix $C_i(q_i, \dot{q}_i)$, the matrix $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew-symmetric, i.e., for a given vector $r \in \mathbb{R}^m$, it follows that*

$$r^T\Big(\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)\Big)r = 0. \tag{4}$$

Property 4. *Consider a mechanical system of the form (1). If $q_i, \dot{q}_i \in \mathbb{L}_\infty$, the Coriolis matrix satisfies $|C_i(q_i, \dot{q}_i)| \leq k_c|\dot{q}_i|$.*

2.2. Graph theory

The information exchange between agents in a multi-agent system can be modeled using graphs. Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ be an graph of order n with the set of nodes $\mathcal{V}(\mathcal{G}) = \{v_1, v_2, \ldots, v_n\}$, the set of edges $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = \{a_{ij}\}$ with nonnegative adjacency elements a_{ij}. A graph is undirected if edges $(i, j) \in \mathcal{E}$ are an unordered pair. In this paper, we assume the graph is undirected. The node indices belong to a finite index set $\mathcal{I} = \{1, 2, \ldots, n\}$. An edge of $\mathcal{G}$ is denoted by $\varepsilon_{ij} = (v_i, v_j)$ and it is said to be incoming with respect to v_j and outgoing with respect to v_i. For an undirected graph, $\forall i, j \in \mathcal{I}$, if $(v_i, v_j) \in \mathcal{E}(\mathcal{G})$, then $(v_j, v_i) \in \mathcal{E}(\mathcal{G})$. The set of neighbors of node v_i is the set of all nodes which communicate to v_i, denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\}$. The graph adjacency matrix $\mathcal{A} = [a_{ij}]$, $\mathcal{A} \in \mathbb{R}^{n\times n}$, is such that $a_{ij} = 1$ if $\varepsilon_{ij} \in \mathcal{E}$ and $a_{ij} = 0$ if $\varepsilon_{ij} \notin \mathcal{E}$. The in-degree of vertex v_i is denoted by $d_i^{in} = \sum_i^n a_{ji}$, and the out-degree of vertex v_i is denoted by $d_i^{out} = \sum_i^n a_{ij}$. If $d_i^{out} = d_i^{in}$ for all $\mathcal{V}(\mathcal{G})$, then the graph is said to be balanced. $\mathcal{D} = \{d_{ij}\} \in \mathbb{R}^{n\times n}$ is called the in-degree or out-degree given as $d_{ii} = d_i^{in}$ or $d_{ii} = d_i^{out}$, $d_{ij} = 0$ and $i \neq j$.

To describe the switching connected topologies, we need to consider all the possible graphs $\{\mathcal{G}_p : p \in \mathcal{P}\}$, where $\mathcal{P}$ is an index set for all graphs defined on node set $\{1, 2, \ldots, n\}$. Define a switching signal $\varrho(t) : [0, \infty) \mapsto \mathcal{P}$ whose value at time t is the index of the graph at time t. Note that all adjacency matrix $\mathcal{A}$ and the Laplacian matrix $\mathcal{L}$ are time varying. Suppose that there is an infinite sequence of bounded, contiguous time-intervals $[t_i, t_{i+1})$, $i = 0, 1, \ldots$ and there is a dwell time $\gamma > 0$, such that $t_{i+1} - t_i \geq \gamma$.

To model the jointly-connected topologies, we consider an infinite sequence of continuous, bounded, non-overlapping time intervals $[t_k, t_{k+1})$, $k = 0, 1, 2, \ldots$ with $t_0 = 0$, $T_0 \leq t_{k+1} - t_k \leq T$ for some constants T_0 and T. Assume that each interval $[t_k, t_{k+1})$ is composed of the following non-overlapping subintervals

$$[t_k^0, t_k^1), \ldots, [t_k^{j-1}, t_k^j), \ldots, [t_k^{m_k-1}, t_k^{m_k})$$

with $t_k^0 = t_k$ and $t_k^{m_k} = t_{k+1}$ for some nonnegative integer m_k. The topology switches at time instants $t_k^0, t_k^1, \ldots, t_k^{m_k}$ which satisfy $t_k^j - t_k^{j-1} \geq \gamma$ (γ is a positive constant) and $0 \leq j \leq m_k$, such that during each subinterval $[t_k^{j-1}, t_k^j)$, the interconnection topology $\mathcal{G}_{\varrho(t)}$ does not change. Note that in each interval $[t_k, t_{k+1})$, $\mathcal{G}_{\varrho(t)}$ is permitted to be disconnected. The graphs are said to be jointly connected across the time interval $[t, t + \mathbb{T}]$, $\mathbb{T} > 0$ if the union of graphs $\{\mathcal{G}_{\varrho(s)} : s \in [t, t + \mathbb{T}]\}$ is connected [8].

2.3. Instrumental lemmas

Lemma 1 *([29]). Define $\epsilon(t) = x_d(t) - x(t)$, $\dot{x}_r(t) = \dot{x}_d(t) + \Gamma\epsilon(t)$, $r(t) = \dot{x}_r(t) - \dot{x}(t) = \dot{\epsilon}(t) + \Gamma\epsilon(t)$, where $x_d(t), x(t) \in \mathbb{R}^m$, $\Gamma \in \mathbb{R}^{m\times m}$ is a positive definite matrix. Let $\epsilon(t) = h(t) * r(t)$, where $*$ denotes the convolution product and $h(t) = L^{-1}(H(s))$ with $H(s)$ being an $m \times m$ strictly proper, exponentially stable transfer function, L^{-1} denotes the inverse transformation of the Laplace manipulator. Then, $r \in \mathbb{L}_2$ implies that $\epsilon \in \mathbb{L}_2 \cap \mathbb{L}_\infty$, $\dot{\epsilon} \in \mathbb{L}_2$, ϵ is continuous and $|\epsilon(t)| \to 0$ as $t \to \infty$. Besides, if $|r(t)| \to 0$ as $t \to \infty$, then $|\dot{\epsilon}(t)| \to 0$.*

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