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Quantization effects on synchronized motion of teams of mobile agents with second-order dynamics

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ABSTRACT

For a team of mobile agents governed by second-order dynamics, this paper studies how different quantizers affect the performances of consensus-type schemes to achieve synchronized collective motion. It is shown that when different types of quantizers are used for the exchange of relative position and velocity information between neighboring agents, different collective behaviors appear. Under the chosen logarithmic quantizers and with symmetric neighbor relationships, we prove that the agents' velocities and positions get synchronized asymptotically. We show that under the chosen symmetric uniform quantizers and with symmetric neighbor relationships, the agents' velocities converge to the same value asymptotically while the differences of their positions converge to a bounded set. We also show that when the uniform quantizers are not symmetric, the agents' velocities may grow unboundedly. Through simulations we present richer undesirable system behaviors when different logarithmic and uniform quantizers are used. Such different quantization effects underscore the necessity for a careful selection of quantization strategies, especially for multi-agent systems with higher-order agent dynamics.

1. Introduction

Recently significant research efforts have been made to study how to coordinate the motion of teams of mobile autonomous agents [1]. One popular approach is to use consensus-type algorithms to guide a team of agents to coincide with one another moving with the same velocity under the conditions that the relative position and/or relative velocity information is shared locally among agents and no agent is isolated from the rest of the team [2–4]. Since agents might be constrained by their limited sensing capabilities, they sometimes cannot acquire their neighboring agents' information through realtime sensing, but rely on digital communication to obtain the needed information in its quantized form. This has motivated a growing number of research activities studying how to design effective coordination control strategies using quantized information [5–12].

Agents governed by second-order dynamics as double-integrators are widely used for modeling mobile autonomous agents especially when the research focus is on the collective team dynamics instead of detailed individual agent dynamics [13]. Multi-agent systems with second-order agent dynamics can have dramatically different collective behavior than those with first-order agent dynamics even when agents are coupled together in similar manners [14]. However, while various quantized consensus schemes have been proposed for multi-agent systems with first-order dynamics [7,10], less is known about the quantization effects on the consensus-type algorithms for motion coordination in systems with higher-order dynamics. Recently some interesting sufficient and/or necessary conditions have been constructed for synchronizing coupled double integrators without quantization [13,14]. In a more recent paper [15], higher-order passive nonlinear systems under quantized measurements are considered, but the coordination task considered there is different and its results cannot be applied directly to the problem considered here.

In this paper, we utilize the control laws that have been used in [13], but study their performances when quantized information is used. Then a new set of tools including new forms of Lyapunov functions are developed accordingly to deal with the challenges in analysis for the discontinuity on the right-hand side of the system equations as a result of quantization. We find in this paper that when the chosen logarithmic quantizers are used in the proposed coordination scheme and the neighbor relationships are symmetric, the agents' velocities and positions get synchronized asymptotically. When the chosen symmetric uniform quantizers are used instead, the agents' velocities converge to the same value asymptotically, while the differences of the agents' positions converge to a bounded set as time goes to infinity; in comparison, when the uniform quantizers are asymmetric, the agents' velocities might

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keep increasing and become unbounded. We also indicate through simulations that richer undesirable system behavior may appear under the chosen uniform and logarithmic quantizers, e.g. the agents' positions may never become the same. Some of such undesirable behaviors are inherently associated with the higher-order agent dynamics. Hence, it is emphasized that when choosing quantization schemes for agents with higher-order dynamics, in order to achieve desired motion coordination, appropriate quantizers have to be picked carefully.

The rest of the paper is organized as follows. In Section 2, the quantized control for motion synchronization is discussed for systems of agents governed by second-order dynamics and the uniform and logarithmic quantizers are defined. We review briefly in Section 3 the tools from nonsmooth analysis that we use. The analysis for systems with the chosen logarithmic and uniform quantizers are discussed in Sections 4 and 5 respectively. We provide some additional simulation results in Section 6 for the case when the neighbor relationships are not symmetric.

2. Motion coordination for agents with second-order dynamics

We consider a team of $N > 0$ autonomous agents, each of which is governed by the following second-order dynamics

$$\begin{cases} \dot{r}_i = v_i \\ \dot{v}_i = u_i \quad i = 1, \dots, N, \end{cases} \tag{1}$$

where $r_i, v_i \in \mathbb{R}^n$ denote the position and the velocity of agent i respectively and u_i is agent i's control input. The goal for designing distributed control laws u_i is to synchronize the motions of the N agents in such a way that the velocities and positions of all the agents become the same asymptotically and thus they move together as a single entity. Such a motion coordination problem has been studied before [13,14], and the solution that has been proposed is to use a consensus-type scheme

$$u_i = -\sum_{j\in\mathcal{N}_1(i)} (r_i - r_j) - \sum_{j\in\mathcal{N}_2(i)} (v_i - v_j), \tag{2}$$

where $\mathcal{N}_1(i)$ (resp. $\mathcal{N}_2(i)$) denotes the set of agent i's neighbors in the graph $\mathbb{G}_1$ (resp. $\mathbb{G}_2$) that describes the neighbor relationships in terms of whether the position (resp. velocity) information can be exchanged between a pair of agents. We use a_{ij} and b_{ij}, $1 \le i, j \le N$, to denote the elements of the adjacency matrices [16] of $\mathbb{G}_1$ and $\mathbb{G}_2$ respectively; in other words, a_{ij} (resp. b_{ij}) equals one if j is a neighbor of i in $\mathbb{G}_1$ (resp. $\mathbb{G}_2$) and zero otherwise. And we set $a_{ii} = 0$, $b_{ii} = 0$ for all $i = 1, \dots, N$.

In the sequel, we assume that $\mathbb{G}_1$ and $\mathbb{G}_2$ are undirected and fixed. Note that in the context of distributed control, each agent only knows the *relative* position or velocity information, i.e. no global coordinate system is available. It has been shown in [13] that when $\mathbb{G}_1$ and $\mathbb{G}_2$ are connected, the control law (2) can achieve the goal effectively.

In this paper, we consider the scenario where for each agent, the relative position and velocity information of its neighbors is acquired through digital communication. Hence, if we continue to use the consensus-type coordination strategy (2), we have the control signals in the following form

$$u_i = -\sum_{j\in\mathcal{N}_1(i)} \mathbf{q}(r_i - r_j) - \sum_{j\in\mathcal{N}_2(i)} \mathbf{q}(v_i - v_j), \tag{3}$$

where $\mathbf{q} : \mathbb{R}^n \to \mathbb{R}^n$ denotes the vector quantizer of choice. Here we have assumed that all the agents have been installed with identical quantizers.

Remark 1. In the literature, when quantizers are applied to agents with *first-order* dynamics, different information has been quantized. For example, in [6] the quantization takes place after the relative positions have been summed up, namely

$$u_i = -\mathbf{q}\left(\sum_{j\in\mathcal{N}_1(i)} (r_i - r_j)\right);$$

in [10] the *absolute* position information in some global coordinate system is quantized, namely

$$u_i = -\sum_{j\in\mathcal{N}_1(i)} \Big(\mathbf{q}(r_i) - \mathbf{q}(r_j)\Big).$$

In [17], the relative position information is quantized in a similar way for what we have done in (3) for second-order agent dynamics. But the coordination task is different, and thus the control goal is different.

In this paper, we consider the following three types of quantizers. The *symmetric uniform quantizer* we consider is a map $q_u : \mathbb{R} \to \mathbb{R}$ such that

$$q_u(x) = \delta_u \left(\left\lfloor \frac{x}{\delta_u} \right\rfloor + \frac{1}{2} \right), \tag{4}$$

where δ_u is a positive number and $\lfloor a \rfloor$, $a \in \mathbb{R}$, denotes the greatest integer that is less than or equal to a. The uniform quantizer (4) is similar to those used in [8,17].

The *asymmetric uniform quantizer* we consider [18] is a map $q_u^* : \mathbb{R} \to \mathbb{R}$ such that

$$q_u^*(x) = \delta_u \left\lfloor \frac{x}{\delta_u} \right\rfloor. \tag{5}$$

The *logarithmic quantizer* we use [8] is an odd map $q_l : \mathbb{R} \to \mathbb{R}$ such that

$$q_l(x) = \begin{cases} e^{q_u(\ln x)} & \text{when } x > 0; \\ 0 & \text{when } x = 0; \\ -e^{q_u(\ln(-x))} & \text{when } x < 0. \end{cases} \tag{6}$$

Note that for the uniform quantizers, the quantization error is always bounded by δ_u, namely $|q_u(x) - x| \le \delta_u$ or $|q_u^*(x) - x| \le \delta_u$ for all $x \in \mathbb{R}$. Note also that for the logarithmic quantizer, it holds that

$$x\, q_l(x) \ge 0, \quad \text{for all } x \in \mathbb{R}, \tag{7}$$

and the equality sign holds if and only if $x = 0$; the quantization error for the logarithmic quantizer is bounded by $|q_l(x) - x| \le \delta_l |x|$, where the parameter δ_l is determined by $\delta_l = 1 - e^{-\delta_u}$.

The above definitions of scalar-valued uniform and logarithmic quantizers can be easily generalized to their counterparts of vector-valued quantizers. Take the logarithmic quantizer as an example. For any $x = \begin{bmatrix} x_1 & \dots & x_n \end{bmatrix}^T \in \mathbb{R}^n$, we define the vector logarithmic quantizer $\mathbf{q}_l(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ to be $\mathbf{q}_l(x) \triangleq \begin{bmatrix} q_l(x_1) & \dots & q_l(x_n) \end{bmatrix}^T$. One can easily check that $\langle x, \mathbf{q}_l(x) \rangle \ge 0$ and the equality sign holds if and only if $x = 0$.

The main result of the paper is to show different quantization effects on the performances of the consensus-type coordination algorithms (3). Because of the discontinuity of the quantized signals, we will make use of nonsmooth analysis of differential equations to solve our problem. We give some preliminaries on nonsmooth analysis in the next section.

3. Preliminaries on nonsmooth analysis

For a differential equation

$$\dot{x}(t) = X(x(t)) \tag{8}$$

where $X : \mathbb{R}^d \to \mathbb{R}^d$ is measurable but discontinuous, the existence of a continuously differentiable solution is not guaranteed. In this paper, we adopt the Filippov solution [19].