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A simple approach for switched control design with control bumps limitation

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ABSTRACT

By its own nature, control of switched systems in general leads to expressive discontinuities at switching times. Hence, this class of dynamic systems needs additional care as far as implementation constraints such as for instance control amplitude limitation is concerned. This paper aims at providing numerically tractable conditions to be incorporated in the control design procedure in order to reduce control bumps. The switching strategy and continuous control laws are jointly determined as well as an $\mathcal{H}_\infty$ guaranteed cost is minimized. Due to its theoretical and practical importance, special attention is given to the dynamic output feedback control design problem. The results are illustrated by means of examples borrowed from the literature which are also used for comparisons that put in evidence the efficiency of the proposed strategy.

1. Introduction

In the context of switched systems, switching plays a major role for stability and performance properties [1,2]. Indeed, these systems are generally controlled by switched controllers and the control signal is intrinsically discontinuous. As far as optimality is concerned, there is no limitation on the amplitude of these discontinuities and one may obtain an optimal switched controller leading to important control discontinuities. These control bumps are not acceptable in a practical situation. In the case of linear time invariant systems, control bumps limitation is known as the "bumpless transfer" problem. At the very beginning, this problem has been formulated for an LTI system as the ability to switch between manual and automatic control while retaining a smooth control signal [3]. "Bumpless transfer" refers to reducing the transients due to switching and much research has been done in the LTI context. This research goes beyond removing input discontinuities and deals with controller re-initialization, addition of extra dynamics, etc. A variety of bumpless transfer methods have been suggested over the years since the 1980s (see [4–12]). In the context of switching systems there are few results which are applicable and our work follows the intuition (confirmed by the examples) that bumps may be reduced by reducing the input discontinuity.

To our knowledge, there are only three approaches in the literature where the bumpless problem is formulated in the context of switched systems. The problem has been first discussed in [13] noticing that switching a hybrid linear controller with gains determined using a standard approach will generally be bumpy. To avoid this, the authors show that if the output matrices of the individual controllers are of full rank, one can non-uniquely determine nonsingular switching conversion matrices and a fixed matrix such that the output matrix of the hybrid controller is constant. The second approach has been proposed recently in [14]. The authors propose a characterization of the controllability of switched linear systems via bumpless transfer inputs and constrained switching. They propose an open loop strategy to design the control input and the switching law such that, at the time of switching, the pre-active and post-active controllers produce control input signals which are as close as possible so as to reduce the magnitude of the control discontinuity. The third approach is presented in [15]. The proposed solution is based on the extension of the LQ optimization theory that has been introduced for the LTI bumpless transfer [10]. To reduce control signal discontinuities, the plant input is forced to follow a target profile for a given period of time using an additional loop which consists of a LQ bumpless transfer controller that is activated at every switching time.

Despite the interest of these approaches, the problem is not completely solved for switched systems and new strategies are more than welcome. Indeed, the limitation of the solution proposed in [13] is related to the fact that it assumes a slow switching configuration as stability of the closed-loop switched

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system is only analyzed by checking if the closed-loop eigenvalues are located in the negative real half of the complex plane. The approach of [14] is an open loop approach and hence the system performance might be sensitive to uncertainties and perturbations as noticed by the authors in the conclusion of this paper. The main drawback of the approach proposed in [15] is related to the fact that the strategy introduces an additional loop even though stability of the closed-loop system is guaranteed by LMI conditions and a dwell time assumption. Here, we propose bumps limitation constraints for switched systems in terms of linear matrix inequalities that can be incorporated in available design strategies. Both stability and performance strategies are concerned. In the case of performance, we propose to use these constraints as a secondary objective to be satisfied. This can be achieved using the notion of the ϵ-suboptimal set of an optimization problem [16]. The proposed strategy allows us to select among the nearly optimal controllers the one leading to limited control discontinuities at the switching instants. The paper is organized as follows. In the next section, we formulate the problem discussed in this paper that is adding bumps limitation constraints in the design of switched control laws for switched systems. Section 3 is dedicated to the derivation of bumps limitation constraints in terms of linear matrix inequalities. These inequalities are used in Section 4 to design nearly optimal switched output feedback controllers that are not bumpy. Several examples are presented in Section 4 to illustrate the efficiency of the proposed method.

Notations. As usual, for real matrices or vectors ($'$) indicates transpose. The symbol ($\bullet$) denotes generically each of its symmetric blocks. The set $\mathcal{M}_c$ consists of all matrices $\Pi \in \mathbb{R}^{N\times N}$ with nonnegative off-diagonal elements satisfying the normalization constraints $\pi_{ii} \leq 0, \sum_{j=1}^{N} \pi_{ji} = 0$ for all $i = 1, \ldots, N$. The squared norm of a trajectory $\xi(t)$ defined for all $t \geq 0$, denoted by $\|\xi\|_2^2$, is equal to $\|\xi\|_2^2 = \int_0^\infty \xi(t)'\xi(t)dt$. The set of all trajectories such that $\|\xi\|_2^2 < \infty$ is denoted by $\mathcal{L}_2$. For real matrices, the Hermitian operator $\mathrm{H_e}\{\cdot\}$ is defined as $\mathrm{He}\{M\} = M + M'$ and the norm $\|M\|_\infty$ equals the maximum singular value of M.

2. Problem formulation

Consider a switched linear system, evolving from zero initial conditions, given by:

$$\dot{x}(t) = A_\sigma x(t) + B_\sigma u(t) + H_\sigma w(t) \tag{1}$$

$$y(t) = C_\sigma x(t) + D_\sigma w(t) \tag{2}$$

$$z(t) = E_\sigma x(t) + F_\sigma u(t) + G_\sigma w(t) \tag{3}$$

where the vectors $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $w \in \mathbb{R}^q$, $z \in \mathbb{R}^p$ and $y \in \mathbb{R}^r$ are the state, the control input, the external disturbance such that $w \in \mathcal{L}_2$, the controlled output and the measured output, respectively. The switching signal is a function $\sigma(t) : t \geq 0 \to \mathbb{K} = \{1, \ldots, N\}$ that selects at each instant of time a particular subsystem among N available ones defined by

$$\mathcal{G}_i = \begin{bmatrix} A_i & B_i & H_i \\ C_i & 0 & D_i \\ E_i & F_i & G_i \end{bmatrix}, \quad i \in \mathbb{K} \tag{4}$$

where the matrices of each state space realization $\mathcal{G}_i$ have compatible dimensions. Different performance criteria can be formulated. In this paper, without loss of generality, we consider the criteria defined by the inequality

$$\sup_{w\in\mathcal{L}_2} \|z\|_2^2 - \rho\|w\|_2^2 < 0 \tag{5}$$

where this criterion has to be understood as follows. If $\sigma(t)$ is a disturbance, the supremum holds for all $\sigma \in \mathcal{S}$ where $\mathcal{S}$ denotes the set of all admissible switching rules; if $\sigma(t)$ is a control variable, then the designed stabilizing switching rule satisfies (5). Notice that whenever the switching rule is kept constant, that is, $\sigma(t) = i \in \mathbb{K}$ for all $t \geq 0$, the quantity ρ equals the standard $\mathcal{H}_\infty$ squared norm of the ith closed-loop subsystem transfer function from the exogenous input w to the controlled output z. The contribution of this paper is not restricted to this type of performance and it can be used without any difficulty for other performance based problems. The bumps limitation problem discussed in this paper is related to the continuous input $u(t)$. Therefore, only strategies proposed in the literature to solve problems where the continuous input $u(t)$ is a design variable are considered:

- *Continuous control design.* The control variable is the continuous input $u(t)$ and stability and performance properties must be acceptable for all admissible switching rules.
- *Joint design.* Both the continuous input $u(t)$ and the switching signal $\sigma(t)$ are control variables to be designed from the available measurements in order to guarantee stability and improve performance [17,18].

Two possible controller structures are considered:

- *State feedback.* The switched controller is characterized by $u(t) = K_\sigma x(t)$, where $K_i, i \in \mathbb{K}$ are constant matrices to be designed.
- *Dynamic output feedback.* The switched controller is characterized by the full order state space equations

$$\dot{\hat{x}}(t) = \hat{A}_\sigma \hat{x}(t) + \hat{B}_\sigma y(t) \tag{6}$$

$$u(t) = \hat{C}_\sigma \hat{x}(t) \tag{7}$$

evolving from the rest, where $\hat{x} \in \mathbb{R}^n$ is the controller state variable and the matrices $(\hat{A}_i, \hat{B}_i, \hat{C}_i)$ for all $i \in \mathbb{K}$ are of compatible dimensions.

For these two structures, σ is considered as a perturbation in the continuous control design problem and it is a control variable to be determined in the joint design problem. A problem of interest concerns the design of these switched linear controllers to make the origin $x = 0$ of the closed-loop system (1)–(3) a globally asymptotically stable equilibrium point assuring that the performance constraint (5) holds for a given parameter $\rho > 0$. The solutions proposed in the literature for these problems lead to controller parameters that are expressed as a function of the Lyapunov matrices to be computed solving convex or non-convex optimization problems. For the state feedback case, the matrices $K_i, i \in \mathbb{K}$ are generally given by

$$K_i = R_i S_i \tag{8}$$

where S_i and R_i are matrices that are obtained by solving matrix inequality based conditions, see [19]. S_i are symmetric positive definite matrices directly linked to the used Lyapunov function. For the output feedback case, the full order dynamic output feedback switched controller has a state space realization (6)–(7) given by

$$\hat{A}_i = V^{-1}(M_i - YA_iX_i - YB_iW_i - L_iC_iX_i)(I - YX_i)^{-1}V \tag{9}$$

$$\hat{B}_i = V^{-1}L_i \tag{10}$$

$$\hat{C}_i = W_i(I - YX_i)^{-1}V \tag{11}$$

where M_i, W_i, L_i, X_i and Y are matrices that are obtained by solving matrix inequality based conditions. X_i and Y are symmetric positive definite matrices directly linked to the used Lyapunov function and V is an arbitrary nonsingular matrix. Without taking into account the limitation of control discontinuities, the obtained controllers may then be bumpy. Our objective in this paper is to propose additional constraints on the controller parameters that take into account the bumps limitation objective.