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Cooperative output regulation of linear multi-agent systems by output feedback[☆]

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ABSTRACT

In this paper, following our recent result on the cooperative output regulation of linear multi-agent systems by a distributed full information state feedback control, we further study the same problem by a distributed measurement output feedback control under certain detectability assumptions. As the problem can be viewed as an extension of the leader-following consensus problem of the linear multi-agent systems, our result contains some existing results on the multi-agent system control as special cases.

1. Introduction

In this paper, we will consider the cooperative output regulation of a group of systems of the following form:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i v, \\ e_i &= C_i x_i + D_i u_i + F_i v, \\ y_{mi} &= C_{mi} x_i + D_{mi} u_i + F_{mi} v, \quad i = 1, \dots, N, \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $e_i \in \mathbb{R}^{p_i}$, $y_{mi} \in \mathbb{R}^{p_{mi}}$, and $u_i \in \mathbb{R}^{m_i}$ are the state, regulated output, measurement output and control input of the ith subsystem. The exogenous signal $v \in \mathbb{R}^q$ represents the reference input to be tracked or the disturbance to be rejected, and is assumed to be generated by a so-called exosystem whose model is given by

$$\dot{v} = S v. \tag{2}$$

The system composed of (1) and (2) can be viewed as a multi-agent system with the exosystem as a leader system, and the N subsystems of (1) as N followers of the leader. For $i = 1, \dots, N$, depending whether or not the measurement output y_{mi} depends on v, or what is the same, whether or not $F_{mi} = 0$, the N followers can be classified into two groups. Without loss of generality, we assume that the first group is composed of the subsystems i, $i = 1, \dots, l$, for some $1 \le l < N$, such that $F_{mi} \neq 0$, and the second group is composed of the subsystems i, $i = l+1, \dots, N$, such that $F_{mi} = 0$. The subsystems in the first group are called the informed followers, while the subsystems in the second group are called the uninformed followers.

A special case of system (1) is $y_{mi} = \mathrm{col}(x_i, v)$,[1] $i = 1, \dots, l$, and $y_{mi} = x_i$, $i = l+1, \dots, N$. The cooperative output regulation problem for such special case was studied by full information control in [1]. Here, by full information control, we mean that the control law can make use of both the state of the plant and the state of the exosystem. As explained in [1], under Assumptions 1–3 to be given in Section 2, if, for each subsystem i, the control u_i can make use of both x_i and v, then we can find a full information control law to solve the output regulation problem of the ith subsystem of (1), and therefore, these N full information control laws together will solve the output regulation problem of system (1) in the classical sense. Such a control scheme was called the decentralized full information control in [1]. Since, for the uninformed followers, the control u_i cannot access the exogenous signal v, the output regulation problem of system (1) cannot be solved, in general, by the decentralized full information control. That is why a distributed full information control scheme was proposed in [1] which solved the output regulation problem of system (1). In many practical situations, neither the state of the plant nor the state of the exosystem is directly available for feedback control. Thus, in this paper, we will further consider the more realistic task of designing

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[1] The symbol $\mathrm{col}(A_1, \dots, A_n)$ denotes $[A_1^T, \dots, A_n^T]^T$ for some given matrices $A_i \in \mathbb{R}^{n_i \times m}$, $i = 1, \dots, n$.

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the output feedback control law to solve the cooperative output regulation problem of the system (1).

As we pointed out in [1], the problem formulated in [1] can be seen as an extension of the work in [2–4]. However, both the papers [2,4] considered the full information case and they assumed all subsystems are identical. The paper [3] assumed that the digraph cannot contain a cycle, and relies on the transmission zero condition which cannot be satisfied if $p_i > m_i$. Another motivation of this problem as shown in [1] is that it includes some existing leader-following consensus problems of multi-agent systems [5–8] as the special cases of our problem.

The rest of this paper is organized as follows. In Section 2 we give a precise description of the linear cooperative output regulation problem. In Section 3 we present our main result. Then we give an example to illustrate our design in Section 4. Finally, in Section 5 we conclude this paper.

Throughout this paper, we use the following notation: $\otimes$ denotes the Kronecker product of matrices. The following properties of the Kronecker product are useful in this paper: $(A\otimes B)(C\otimes D) = (AC)\otimes(BD)$, $(A+B)\otimes C = A\otimes C + B\otimes C$, and $A\otimes(B+C) = A\otimes B + A\otimes C$. The symbol $\mathbb{1}_N$ denotes a N dimensional column vector with all elements 1. The symbol $0_{m\times n}$ denotes the zero matrix in $\mathbb{R}^{m\times n}$. The symbol block diag$(A_1,\ldots,A_N)$ denotes the block diagonal matrix whose ith diagonal block is $A_i \in \mathbb{R}^{n_i\times n_i}$, $i = 1,\ldots,N$. The symbol diag$(a_1,\ldots,a_N)$ denotes the diagonal matrix whose ith diagonal element is $a_i \in \mathbb{R}$.

2. Assumptions and problem statement

Let us first list some assumptions as follows.

Assumption 1. S has no eigenvalues with negative real parts.

Assumption 2. The pairs (A_i, B_i) are stabilizable, $i = 1,\ldots,N$.

Assumption 3. For every $i = 1,\ldots,N$, the linear matrix equations

$$\begin{aligned} X_iS &= A_iX_i + B_iU_i + E_i,\\ 0 &= C_iX_i + D_iU_i + F_i, \end{aligned} \tag{3}$$

have a solution (X_i, U_i).

Assumption 4. The pairs $\left(\begin{bmatrix} C_{mi} & F_{mi}\end{bmatrix}, \begin{bmatrix} A_i & E_i\\ 0 & S\end{bmatrix}\right)$ are detectable, $i = 1,\ldots,l$.

Assumption 5. The pairs (C_{mi}, A_i) are detectable, $i = l+1,\ldots,N$.

Remark 1. Assumptions 1–3 are standard ones in the output regulation literature [9–12]. In particular, Eqs. (3) are called regulator equations whose solvability is a necessary condition for the output regulation problem. It is known that the solvability of the regulator Eqs. (3) is guaranteed if

$$\operatorname{rank}\begin{bmatrix} A_i - \lambda I & B_i\\ C_i & D_i\end{bmatrix} = n_i + p_i, \quad \forall \lambda \in \sigma(S), \tag{4}$$

where $\sigma(S)$ denotes the spectrum of S. Condition (4) is known as the transmission zero condition. However, the transmission zero condition is not necessary for the solvability of the regulator equations as indicated in Remark 1.12 of [12]. For example, when $p_i > m_i$, the transmission zero condition fails. However, the regulator Eqs. (3) may still admit a solution pair if $\operatorname{vec}\left(\begin{bmatrix} E_i\\ F_i\end{bmatrix}\right)$ is in the range of the matrix $S^T \otimes \begin{bmatrix} I_{n_i} & 0_{n_i\times m_i}\\ 0_{p_i\times n_i} & 0_{p_i\times m_i}\end{bmatrix} - I_q \otimes \begin{bmatrix} A_i & B_i\\ C_i & D_i\end{bmatrix}$.

Remark 2. Assumption 4 means that, for the informed followers, the plant state x_i, $i = 1,\ldots,l$, and the exogenous signal v, are detectable from the measurement output y_{mi}. Assumption 5 means that, for the uninformed followers, the plant state x_i, $i = l+1,\ldots,N$, is detectable from the measurement output y_{mi}, but the exogenous signal v is not detectable from the measurement output y_{mi}. Thus, the output regulation problem of system (1) cannot be solved, in general, by a decentralized output feedback control scheme. Therefore, we will employ a distributed dynamic measurement output feedback controller to solve our problem.

To introduce our control law, let us first define a Metzler matrix[2] with zero row sum $M = [a_{ij}] \in \mathbb{R}^{N\times N}$, $i, j = 1,\ldots,N$. Then, we consider the following distributed dynamic measurement output feedback controller:

$$u_i = K_{1i}\xi_i + K_{2i}\eta_i, \quad i = 1,\ldots,N,$$

$$\begin{cases} \text{If } i = 1,\ldots,l,\\ \begin{bmatrix}\dot\xi_i\\ \dot\eta_i\end{bmatrix} = \begin{bmatrix} A_i & E_i\\ 0 & S\end{bmatrix}\begin{bmatrix}\xi_i\\ \eta_i\end{bmatrix} + \begin{bmatrix} B_i\\ 0\end{bmatrix}u_i\\ \quad + \begin{bmatrix} L_{1i}\\ L_{2i}\end{bmatrix}(C_{mi}\xi_i + D_{mi}u_i + F_{mi}\eta_i - y_{mi}) \end{cases} \tag{5}$$

$$\begin{cases} \text{If } i = l+1,\ldots,N,\\ \dot\xi_i = A_i\xi_i + B_iu_i + E_i\eta_i + L_i(C_{mi}\xi_i + D_{mi}u_i - y_{mi})\\ \dot\eta_i = S\eta_i + \mu\sum_{j=1}^{N} a_{ij}(\eta_j - \eta_i) \end{cases}$$

where μ is some positive number, $K_{1i} \in \mathbb{R}^{m_i\times n_i}$, $K_{2i} \in \mathbb{R}^{m_i\times q}$, $L_{1i} \in \mathbb{R}^{n_i\times p_i}$, $L_{2i} \in \mathbb{R}^{q\times p_i}$, and $L_i \in \mathbb{R}^{n_i\times p_i}$ are gain matrices to be described in Remark 4. Then we describe the linear cooperative output regulation problem as follows:

Definition 1. Given the system (1), and exosystem (2), find a control law of the form (5) such that

(1) The system matrix of the overall closed-loop system is Hurwitz.

(2) For any initial condition $x_i(0)$, $\eta_i(0)$, $\xi_i(0)$, $i = 1,\ldots,N$, and $v(0)$, the regulated output

$$\lim_{t\to\infty} e_i(t) = 0, \quad i = 1,\ldots,N.$$

3. Solvability of the problem

Define $\Delta = \operatorname{diag}(a_{10},\ldots,a_{N0})$ with $a_{i0} > 0$ for $i = 1,\ldots,l$, and $a_{i0} = 0$ for $i = l+1,\ldots,N$. Let $H = -M + \Delta$, and

$$\bar{\mathcal{L}} = \left[\begin{array}{c|c} 0 & 0_{1\times N}\\ \hline -[a_{10},\ldots,a_{N0}]^T & H\end{array}\right].$$

Clearly, $-\bar{\mathcal{L}}$ is a Metzler matrix with zero row sum. We now establish the following property for the matrix $\bar{\mathcal{L}}$.

Lemma 1. *Let $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ be the digraph of $\bar{\mathcal{L}}$ with $\bar{\mathcal{V}} = \{0, 1,\ldots,N\}$. Partition the matrix $\bar{\mathcal{L}}$ as follows:*

$$\bar{\mathcal{L}} = \left[\begin{array}{c|cc} 0_{1\times 1} & 0_{1\times l} & 0_{1\times(N-l)}\\ \mathcal{L}_{21} & \mathcal{L}_{22} & \mathcal{L}_{23}\\ \mathcal{L}_{31} & \mathcal{L}_{32} & \mathcal{L}_{33}\end{array}\right]$$

where $\mathcal{L}_{22} \in \mathbb{R}^{l\times l}$, and $\mathcal{L}_{33} \in \mathbb{R}^{(N-l)\times(N-l)}$. Then

$$\mathcal{L}_{32}\mathbb{1}_l + \mathcal{L}_{33}\mathbb{1}_{N-l} = 0. \tag{6}$$

Furthermore, $\mathcal{L}_{33}$ is nonsingular if and only if the digraph $\bar{\mathcal{G}}$ contains a directed spanning tree with node 0 as the root, and if $\mathcal{L}_{33}$ is nonsingular, then all the eigenvalues of $\mathcal{L}_{33}$ have positive real parts.

Proof. Since $a_{i0} = 0$, $i = l+1,\ldots,N$, $\mathcal{L}_{31} = 0$. Thus (6) directly follows from the property $\bar{\mathcal{L}}\mathbb{1}_{N+1} = 0$.

[2] See the Appendix for a self-contained summary of the digraph.