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A new reduced-order observer normal form for nonlinear discrete time systems

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ABSTRACT

This paper presents a new observability normal form for discrete-time nonlinear systems. This form enables us to design a reduced-order observer. Necessary and sufficient geometrical conditions for the existence of a coordinate change to transform a discrete-time nonlinear system into such normal form are given. An illustrative example is given to show the effectiveness of our approach.

1. Introduction

The problem of observing the state variables of a deterministic dynamical system has been the object of numerous studies ever since the original work of Luenberger [1] first appeared. The problem is to fully or partially reconstruct (or to estimate) the system state by using the input and output measurements. Unlike linear systems, observers design for nonlinear systems is not an easy task. The observers design for nonlinear systems has attracted significant attention (see [2–6] and references therein). Observers for continuous time nonlinear systems have been considered recently in [7,8]. One technique for constructing nonlinear observers is to linearize the dynamic error by using a change of coordinates and an output injection (see [9–11,8,12]). Recently, a new approach for the design of nonlinear observers that can be applied to a large class of nonlinear systems has been presented (see [12,13]). Compared to the nonlinear observers design for continuous time systems, only few results exist for the discrete-time systems (see [6]).

In [14], the authors gave the necessary and sufficient conditions for a discrete-time nonlinear system to be equivalent to a nonlinear full order observer normal form. This form is defined as a normal one for which a full order observer can be constructed with linear dynamic error.

On the other hand, reduced-order observers design has involved various researchers in control field. The first results on the reduced order observers for linear systems were presented by Luenberger [1].

In [15–17], the reduced-order observers design for discrete-time nonlinear systems was presented. Recently a reduced-order observer normal form for continuous nonlinear dynamical systems has been given in [18].

In this paper, as in [19], we give the discrete-time version of the reduced observability normal form obtained in the continuous case [18]. Even though normal forms for the continuous and discrete cases are the same, the methods to transform systems into these normal forms are quite different. Indeed in the continuous time case we handle vector fields and in the discrete case we handle maps. This paper gives a generalization of the construction of reduced-order observers for linear systems developed by Luenberger [1]. However, the determination of the normal forms that lead to reduced-order observers with a linear error remains unknown. The main contribution of this paper is to give these normal forms and then to characterize the family of nonlinear dynamical systems which can be transformed by means of a change of coordinates into these forms.

The paper is organized as follows. Section 2 contains notations and a definition. Section 3 presents a nonlinear normal form and the associated reduced-order observer. Section 4 deals with geometrical conditions under which a nonlinear discrete-time dynamic system can be transformed into such a normal form. Section 5 presents an example to illustrate our results, and Section 6 concludes the paper.

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2. Notations and definition

Let us consider the following discrete-time nonlinear dynamical system:

$$x_{k+1} = F(x_k) \tag{1a}$$

$$y_k = h(x_k) \tag{1b}$$

where the $x_k \in \mathbb{R}^n$ is the state, and $y_k \in \mathbb{R}^m$ is the output. We assume that the map F is a diffeomorphism. We assume, also, that the pair (h, F) satisfies the observability rank condition. Thus, the following 1-forms:

$$d(hoF^{i-1}) \quad 1 \le i \le n$$

are linearly independent, where $F^{i-1} = \underbrace{FoF \cdots oF}_{\text{times } (i-1)}$ is the $(i-1)$th composition of F and $d(hoF^{i-1})$ is the differential of the function hoF^{i-1}.

Under this assumption see for e.g. [16] the dynamical system (1) can be rewritten in new coordinates as follows:

$$x^1_{k+1} = f_1(x_k) \tag{2a}$$

$$x^2_{k+1} = f_2(x_k) \tag{2b}$$

$$y_k = x^2_k \tag{2c}$$

where vector $x_k = \begin{pmatrix} x^1_k \\ x^2_k \end{pmatrix} \in \mathbb{R}^n$, denotes the whole state, $x^1_k \in \mathbb{R}^{n-m}$ is the unmeasured state and $x^2_k \in \mathbb{R}^m$ is the measured state (output).

It is easy to see that the pair (h, F) satisfies the observability rank condition if and only if the pair (f_2, f_1) as well satisfies the observability rank condition, where f_2 is considered as an output.

Now, let us define the so-called reduced order observer.

Definition 1. We say that a discrete-time system in the following form

$$\hat{x}^1_{k+1} = \widetilde{f}_1(\hat{x}^1_k, x^2_k) \tag{3}$$

is a reduced order observer for the discrete time dynamical system (2) if the error $e_k = \hat{x}^1_k - x^1_k \longrightarrow 0$ as $k \longrightarrow +\infty$.

In the case where maps f_1 and f_2 are linear, i.e.

$$f_1(x_k) = A_{11}x_{1,k} + A_{12}x_{2,k} \quad \text{and} \quad f_2(x_k) = A_{21}x_{1,k} + A_{22}x_{2,k}$$

then it is easy to see that the reduced order observer is of the following form:

$$\hat{x}^1_{k+1} = A_{11}\hat{x}^1_k + A_{12}x^2_k + K(A_{21}x^1_k - A_{21}\hat{x}^1_k). \tag{4}$$

In fact let $e_k = \hat{x}^1_k - x^1_k$ be the observation error, then its dynamic behaves as follows:

$$e_{k+1} = (A_{11} - KA_{21})e_k.$$

Therefore, there exists a gain matrix K such that eigenvalues of $(A_{11} - KA_{21})$ can be placed arbitrarily as a long as the pair (A_{21}, A_{11}) is observable.

We will end this section by a remark.

Remark 1. The reduced order observer (4) introduced by Luneberger [1] enables us to overcome the redundancy of measurement. However, the new output $A_{21}x^1_k$ contains more information than that we need. For example:

$$x^1_{k+1} = f_1(x^1_k, x^2_k) = \begin{pmatrix} x_{1,k+1} \\ x_{2,k+1} \\ x_{3,k+1} \end{pmatrix} = \begin{pmatrix} Ly_k \\ x_{1,k} \\ x_{2,k} \end{pmatrix}$$

$$x^2_{k+1} = f_2(x^1_k, x^2_k) = \begin{pmatrix} \xi_{k+1} \\ \eta_{k+1} \end{pmatrix} = \begin{pmatrix} x_{3,k} + \xi_k \\ ax_{1,k} + bx_{2,k} + cx_{3,k} \end{pmatrix}$$

$$y_k = x^2_k = \begin{pmatrix} \xi_k \\ \eta_k \end{pmatrix}.$$

Thus

$$A_{21} = \begin{pmatrix} 0 & 0 & 1 \\ a & b & c \end{pmatrix}$$

where L is a constant matrix with appropriate dimension. From the form of the above dynamical system, we can see that if we want to estimate the state x_k, we do not need the last equation η_{k+1}, we only need to use the output ξ_k and its dynamic

$$\xi_{k+1} = x_{3,k} + \xi_k$$

to design a reduced order observer. In the sequel, we will take into account this fact in the proposed normal form.

3. Normal form and its reduced-order observer

In this section, we will give a nonlinear observability normal form which admits a reduced-order observer. We consider a multivariable discrete-time nonlinear system described by

$$z_{k+1} = Az_k + \alpha(y_k)\, z_{0,k} + \beta(y_k) \tag{5a}$$

$$\xi_{k+1} = \gamma_1(y_k)\, z_{0,k} + \gamma_2(y_k) \tag{5b}$$

$$\eta_{k+1} = \mu(z_{0,k}, y_k) \tag{5c}$$

$$y_k = (\xi_k, \eta_k)^T = (y^1_k, y^2_k)^T \tag{5d}$$

where

$$\begin{pmatrix} z^1_{1,k} & \cdots & z^1_{r_1,k} & \cdots & z^m_{1,k} & \cdots & z^m_{r_m,k} \end{pmatrix}^T \in \mathbb{R}^r$$

is the unmeasured state with $r = \sum_{i=1}^m r_i = n - m - p$,

$$\xi_k = \begin{pmatrix} \xi_{1,k} & \cdots & \xi_{m,k} \end{pmatrix}^T \in \mathbb{R}^m,$$

$$\eta_k = \begin{pmatrix} \eta_{1,k} & \cdots & \eta_{p,k} \end{pmatrix}^T \in \mathbb{R}^p$$

are the measured outputs, and

$$z_{0,k} = Cz_k = \begin{pmatrix} z^1_{r_1,k} & z^2_{r_2,k} & \cdots & z^m_{r_m,k} \end{pmatrix}$$

where $C = \operatorname{diag}(C_1, \ldots, C_m)$ with $C_i = \begin{pmatrix} 0 & \cdots & 0 & 1 \end{pmatrix}$ for $i = 1, \ldots, m$, and

$$A = \operatorname{diag}(A_1, \ldots, A_m),$$

where A_i is an $r_i \times r_i$ matrix in the following form:

$$A_i = \begin{pmatrix} 0 & \cdots & 0 & 0 & 0 \\ 1 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \cdots & \vdots \\ 0 & \cdots & 1 & 0 & 0 \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix}.$$

We will assume that $\gamma_1(y_k)$ is an invertible map such that from (5b) we obtain:

$$z_{0,k} = Cz_k = (\gamma_1(y_k))^{-1}(\xi_{k+1} - \gamma_2(y_k)). \tag{6}$$

Therefore the pair (C, A) is observable.

Now, let us consider the following system

$$\zeta_{k+1} = N\zeta_k + \Psi(y_k, y_{k-1}) \tag{7a}$$

$$\hat{z}_k = \zeta_k + G(y_k, y_{k-1}) \tag{7b}$$

where N is a matrix of appropriate dimension, G and Ψ are nonlinear maps which must be determined such that (7) is an asymptotic observer for system (5). One can see that the form (7) is a generalization of the functional and reduced order observers form considered in [20] for example.

We have the following result.

Proposition 1. *Assume that the pair (C, A) is observable, and let κ such that $N = A - \kappa C$ be Hurwitz; then system (7) is an asymptotic*