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Stabilization of non-minimum phase switched nonlinear systems with application to multi-agent systems☆

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ABSTRACT

This paper addresses the stabilization issue of a class of switched nonlinear systems where each mode may be non-minimum phase, and the states of linearized dynamics of all modes compose the whole state space. Time dependent and state dependent stabilization switching laws are provided by considering both common and multiple Lyapunov functions. The new results are applied to the aggregation problem of nonlinear multi-agent systems with designable switching connection topology. Finally, an aircraft team example illustrates the efficiency of the proposed approaches.

1. Introduction

Many engineering applications can be modeled by switched systems due to the existence of various jumping parameters [1]. Fruitful results have been reported on stability and stabilization of switched nonlinear systems, e.g., [2–5] to name a few.

Stabilization of non-minimum phase nonlinear systems is a quite challenging problem. Several fundamental methods have been proposed including state feedback control [6], and output feedback control [7–9]. The main idea behind these methods is to compensate for the unstable zero dynamics by means of output synthesis or auxiliary systems such that the system becomes stable. The controllability of the zero dynamics is the basic requirement, otherwise, the stabilization cannot be achieved.

Some contributions have also been devoted to non-minimum phase switched nonlinear systems where each nonlinear mode may be non-minimum phase. In [10], H_∞ control goal is achieved for a class of non-minimum phase cascade switched nonlinear systems where the internal dynamics of each mode is assumed to be asymptotically stabilizable. Output tracking of non-minimum phase switched nonlinear systems has been considered in [11], where an approximated minimum phase model is utilized. The same problem is also investigated in [12] by means of an inversion-based control strategy.

The main idea of the above results consists of two steps:

1. Design the individual controller respectively in each mode to compensate for its own unstable internal dynamics such that all modes become stable individually. This can be done by using the existing techniques for non-minimum phase nonlinear systems.
2. Apply the standard stability condition of switched systems, e.g. common/multiple Lyapunov functions methods to achieve the stability of the whole switched system.

This idea is natural and extends the approaches of non-switching systems to the switched one. However, it is well known that the stabilization for non-minimum phase nonlinear systems is quite difficult, and is even impossible to be achieved if the unstable zero dynamics is uncontrollable.

In this work, we focus on the stabilization of non-minimum phase switched nonlinear systems where the internal dynamics of each mode may be unstable and uncontrollable. Different from [10–12], we do not try to compensate for the unstable internal dynamics respectively in each mode. Instead, we achieve the stabilization from the overall system point of view. It will show that under some conditions, the negative effects of internal dynamics in some modes may be compensated by other modes, and the overall switching process can still be stable. The similar idea can be seen in [13] for switched linear systems and [14] for switched nonlinear

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systems. This special property of a switched system provides a new clue for stabilizing the non-minimum phase systems. Two benefits follow:

1. It relaxes significantly the condition on the internal dynamics (it is allowed to be unstable and uncontrollable simultaneously).
2. It makes easier the design of each mode's individual controller.

We consider a class of switched nonlinear systems where each mode may be non-minimum phase, and the states of linearized dynamics of all modes compose the whole state space. Consequently, we provide novel time dependent and state dependent stabilization switching laws with considering both common and multiple Lyapunov functions. The proposed results are more general and flexible than that in [14] where only a time-dependent switching law is provided.

As an important application of the new results, we consider the target aggregation problem of nonlinear multi-agent systems which can be divided into several groups of subsystems with each group being in the leader-following structure, and only one group is allowed to be interconnected at one time. Inspired by the linearized feedback control idea [15], we design a novel "feedback control topology" for each group, under which the multi-agent system with switching topologies can be regarded as a non-minimum phase switched system and the proposed switched system results are applied.

In the rest of the paper, some preliminaries are given in Section 2. Sections 3 and 4 propose time dependent and state dependent stabilization switching laws respectively, which are applied to multi-agent systems in Section 5, followed by some concluding remarks in Section 6.

2. Preliminaries

Consider the following switched nonlinear control systems

$$\dot{x} = f_\sigma(x) + g_\sigma(x)u_\sigma \tag{1}$$

where $x \in \Re^n$ are available states. Define $\mathcal{M} = \{1, 2, \dots, m\}$, where m is the number of modes. $\sigma(t) : [0, \infty) \to \mathcal{M}$ denotes the *switching signal*, which is assumed to be a piecewise constant function continuous from the right. $\forall i \in \mathcal{M}$, $u_i \in \Re$ is the input, f_i and g_i are smooth functions with $f_i(0) = g_i(0) = 0$. It is required that the activating period of each mode is not less than τ where $\tau > 0$ is called "dwell-time" [1]. We also assume that the states do not jump at the switching instants.

Suppose that for each mode $i \in \mathcal{M}$, we can find a function y_i and a partition $x = [\bar{x}_i^\top, x_i^\top]^\top$ where $x_i \in \Re^{r_i}$, $\bar{x}_i \in \Re^{n-r_i}$ to rewrite the system (1) into the normal form [15]:

$$\dot{\bar{x}}_i = \psi_i(\bar{x}_i, x_i) \tag{2}$$

$$\dot{x}_i = Ax_i + b_i(\bar{x}_i, x_i) + a_i(\bar{x}_i, x_i)u_i \tag{3}$$

$$y_i = Cx_i \quad i = 1, 2, \dots, m \tag{4}$$

where

$$A_i \in \Re^{r_i \times r_i} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$C_i \in \Re^{1 \times r_i} = \begin{bmatrix} 1, 0, \dots, 0 \end{bmatrix}$$

and $b_i(\bar{x}_i, x_i) = [0, 0, \dots, 0, \bar{b}_i(\bar{x}_i, x_i)]^\top$, $a_i(\bar{x}_i, x_i) = [0, 0, \dots, 0, \bar{a}_i(\bar{x}_i, x_i)]^\top$ with $\bar{b}_i$ and $\bar{a}_i$ being scalar functions, $\bar{a}_i \neq 0$.

Mode i is a *non-minimum phase* if its zero dynamics $\dot{\bar{x}}_i = \psi_i(\bar{x}_i, 0)$ is unstable, otherwise it is a *minimum phase*. The problem to be solved in this paper is *whether the switched system (1) with all non-minimum phase modes satisfying (2)–(4) can be stabilized by using σ and u_σ?*

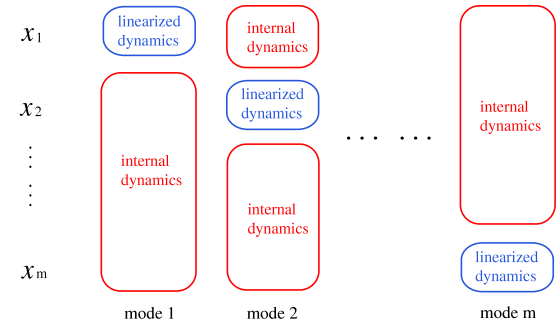

Fig. 1. The structure of switched systems.

A basic assumption throughout the paper is given:

Assumption 2.1. For a switched system (1) with all modes satisfying (2)–(4), it holds that $x = [x_1^\top, \dots, x_m^\top]^\top$. □

Assumption 2.1 implies that the states of unstable internal dynamics in each mode can be controlled and linearized in other modes. The states of all linearized dynamics compose the whole state space as illustrated in Fig. 1. This allows us to achieve the stabilization by fully utilizing the tradeoff among different modes of the switched system. Assumption 2.1 can be relaxed to a more moderate case where more than one mode may share some states in their linearized dynamics. The proposed methods can be straightforwardly extended to this case.

Remark 2.1. We would like to show how to fully utilize the switching properties to achieve the stabilization goal. Therefore, an idea of "overall system point of view" will be followed throughout the paper. For more general switched systems where Assumption 2.1 is not satisfied, the combination between the existing control techniques for non-switching systems and the switching properties can be developed. □

3. Time-dependent switching law

In this section, we provide time-dependent stabilizing switching laws for the switched system satisfying Assumption 2.1.

3.1. Common Lyapunov function

Define a special function that will be used in the following sections:

Definition 3.1. A class $\mathcal{GKL}$ function $\gamma : [0, \infty) \times [0, \infty) \to [0, \infty)$ if $\gamma(\cdot, t)$ is of class $\mathcal{K}$[1] for each fixed $t \geq 0$ and $\gamma(s, t)$ increases to infinity as $t \to \infty$ for each fixed $s \geq 0$. □

Assumption 3.1. For the switched system (1) satisfying Assumption 2.1, we can design u_i for each mode i under which there exists a continuous non-negative function

$$V : \Re^n \to \Re_{\geq 0} = V_1(x_1) + V_2(x_2) + \cdots + V_m(x_m) \tag{5}$$

where $V_k(x_k) \in \mathcal{C}^1 : \Re^{r_k} \to \Re_{\geq 0}$, $k \in \mathcal{M}$, and there exist $\lambda_i > 0$, $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$, $\gamma_{ab} \in \mathcal{K}_\infty$, for $a, b \in \mathcal{M}$, and $\phi_j \in \mathcal{GKL}$ $\forall j \in \mathcal{M} - \{i\}$ such that

[1] Class $\mathcal{K}$ is a class of strictly increasing and continuous functions $[0, \infty) \to [0, \infty)$ which are zero at zero. Class $\mathcal{K}_\infty$ is the subset of $\mathcal{K}$ consisting of all those functions that are unbounded.