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A robust fault detection filter for polynomial nonlinear systems via sum-of-squares decompositions

Giuseppe Franzè *, Domenico Famularo

DEIS — Università degli Studi della Calabria, Rende (CS), 87036, Italy

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ABSTRACT

A novel diagnostic framework is discussed for fault detection of nonlinear systems whose structure is described by multivariate polynomials. The trade-off between disturbance rejection and fault sensitivity prescriptions is characterized via algebraic geometry conditions and the unknown input observer design problem is formulated via sum-of-squares (SOS) technicalities by exploiting the results of the Positivstellensatz Theorem. An adaptive threshold logic is proposed to reduce the generation of false alarms, and the diagnostic filter capabilities are illustrated via a numerical example taken from the literature.

1. Introduction

Fault diagnosis covers an important class of methodologies and instruments in systems engineering to improve plant reliability and tolerance to anomalous and faulty events. Strictly speaking, a fault can be considered as an undesired and unexpected event in the process mainframe which tends to degrade the overall plant performance. Some faults, if not promptly and properly detected, could turn into unrecoverable failures, causing serious damage. By considering that the actual demand for enhanced productivity leads to challenging plant operations, advanced supervision and fault detection (FD) schemes can help to improve the overall plant efficiency and the reconfiguration capabilities of the control laws as well by the early detection and accommodation of system anomalies. From the previous discussion it is clear that the FD issue involves decisions, based on the monitored data, on whether there is a fault or the system is running normally. Many authors have addressed such a crucial point in several books and survey articles by using different approaches (model-based design, parameter estimation, generalized likelihood ratio, etc.). See [1,2] and the references therein for comprehensive and up-to-date tutorials.

Amongst all the existing methodologies, the design and analysis of model-based FD paradigms via the analytical redundancy approach has received significant attention in the last two decades. This approach hinges upon two components, an unknown input observer (UIO) and a decision logic. The UIO role is to simultaneously decouple the residuals from the exogenous disturbances and increase the sensitivity of the residuals with respect to (w.r.t.) faults. The decision logic is in charge instead to possibly discriminate between real and false alarms [3,2].

Robust model-based FD problems have been studied mainly for linear system frameworks, and many effective methods have been developed. For a more thorough review, the reader is referred to [3] and the references therein. However, it must be emphasized that using a linear approach gives rise to an unavoidable level of conservativeness if the system to be monitored is strongly nonlinear and a significant number of working points need to be covered during operations. Then, the development of direct nonlinear FD methods may be of interest and may play a key role in some specific applications: observer-based [4,5] and adaptive threshold approaches [1] have been proposed for a class of Lipschitz nonlinear systems with unstructured modeling uncertainty [6].

Recent advancements in sum-of-squares (SOS) programming techniques and the results of the Positivstellensatz Theorem from real algebraic geometry make it possible to state analysis and feedback design problems for polynomial nonlinear systems as SOS programs which are computationally tractable; see, e.g., [7]. In particular, SOS decomposition problems have been found to be solvable by means of semi-definite programming techniques, whose computational complexity has been shown to be polynomial in the problem size [8]. Applications of polynomial methodologies for FD problems are few in the literature; see, e.g., [9]. In this contribution, an FD observer design method is described by means of homogeneous and generalized Krasovskii-type Lyapunov functionals which are non-quadratic with respect to the control system output.

* Corresponding author.
E-mail addresses: franze@deis.unical.it (G. Franzè), famularo@deis.unical.it (D. Famularo).

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Moving from these considerations, here we propose a novel FD design procedure for nonlinear systems whose vector field is a multivariate polynomial in the state plant components. The proposed filter is designed to jointly decouple the residuals from the disturbances and conversely to enhance the residual sensitivity to faults. The residual generator consists of a "Luenberger-like" unknown nonlinear input observer (UNIO) [4], whose gain is also a multivariate polynomial in the state estimate variables. Rigorous frequency conditions ensuring solvability of the proposed nonlinear FD design problem are derived by resorting to Volterra series expansions of the residual output. The residual's Volterra expansion is instrumental for FD solvability reasons to characterize the frequency separation conditions between the disturbance/reference-input rejection requirements and the fault sensitivity prescriptions [10–12]. It should be noted that, to our best knowledge, the joint use of the frequency decoupling conditions in a nonlinear framework and the proposed SOS-based design are a significant first attempt in literature to characterize the FD UNIO design. Even if the solution gives rise to Bilinear Matrix Inequalities (BMIs), the key aspect consists in an inherent FD framework easiness because the proposed scheme traces out a linear robust frequency-based FD procedure (see [13,2] and references therein for details).

The FD outputs are then evaluated by means of an "rms-norm" time-based function and compared with a non-conservative threshold. The logic is equipped with an adaptive mechanism, whose parameters are computed from the outset in the worst-case scenario. This ensures that, for any nuisance occurrence, the residual error remains below its threshold, avoiding the generation of false alarms. From a computational point of view, such a functional is obtained by jointly bounding the estimation error and solving suitable SOS programming problems to compute the related coefficients.

A simulation example taken from the literature is finally considered in order to illustrate the benefits and the effectiveness of the proposed SOS-based FD scheme.

Notation

- With $\mathbb{R}[x]$ we denote the ring of multivariate scalar polynomials $p \in \mathbb{R}[x]$ in the unknown $x \in \mathbb{R}^n$.
- With $\Sigma[x] \subset \mathbb{R}[x]$ we denote the proper and closed subset

$$\Sigma[x] := \left\{ s \in \mathbb{R}[x] | \exists q < \infty, \exists \{p_i\}_{i=1}^q, p_i \in \mathbb{R}[x], \text{ s.t. } s = \sum_{i=1}^q p_i^2 \right\}$$

  of multivariate SOS polynomials $s \in \Sigma[x]$ in the unknown $x \in \mathbb{R}^n$.
- The polynomial $p(x)$, having degree $2d$, belongs to $\Sigma[x]$ iff there exists a symmetric matrix $Q = Q^T \geq 0$ such that $p(x) = z^T(x)Qz(x)$, $z(x) = [1, x_1, x_2, \ldots, x_n, x_1x_2, \ldots, x_n^d]^T$, with $z(x)$ containing all monomials in the variables $x_1, \ldots, x_n$ of degree lower than or equal to d. The matrix Q is known as the Gram matrix or the square matricial representation (SMR) of $p(x)$ [14,15].
- Given a finite-energy signal $w(\cdot)$, we denote with

$$\Omega_w \triangleq \left\{ w(\cdot) \middle| \exists \varepsilon_w > 0 \text{ s.t. } \sqrt{\int_0^\infty \|w(t)\|_2^2 dt} \leq \varepsilon_w \right\}$$

  the L_2 ball of radius ϵ_w.
- Given a set $S \subseteq X \times Y \subseteq \mathbb{R}^n \times \mathbb{R}^m$, the projection of the set S onto X is defined as $\text{Proj}_X(S) := \{x \in X \mid \exists y \in Y \text{ s.t. } (x, y) \in S\}$.

2. Problem statement

Let us consider a plant described by the following nonlinear model:

$$\begin{cases} \dot{x}(t) = a(x(t)) + b(x(t))u(t) + e(x(t))f(t) + g(x(t))d(t) \\ y(t) = h(x(t)), \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ denotes the state, $u(t) \in \mathbb{R}^m$ the reference input, and $y(t) \in \mathbb{R}^p$ the measured output; $f(t) \in \mathbb{R}^{m_f}$ denotes the fault signal, $d(t) \in \mathbb{R}^{m_d}$ the exogenous disturbance; $a(x) \in \mathbb{R}^n[x]$, $b(x) \in \mathbb{R}^{n\times m}[x]$, $e(x) \in \mathbb{R}^{n\times m_f}[x]$, $g(x) \in \mathbb{R}^{n\times m_d}[x]$ and $h(x) \in \mathbb{R}^p[x]$ are arrays and matrices of multivariate polynomials, respectively. The reference, disturbance, and fault inputs are supposed to belong respectively to the sets Ω_f, Ω_d, and Ω_u.

In what follows, we will assume that 0_x is a zero-input equilibrium and, for conciseness, we will limit our attention to process disturbances and actuator faults. This is not a serious limitation, because the presence of sensor faults and noises can be easily addressed by recasting them as actuator faults and process disturbances; see [3]. Moreover, the polynomial system (1) is assumed to be observable, see [16], and this structural condition can be tested via simple SOS procedures based on Gramians arguments as outlined in [17].
Based on the system representation (1), the objective is to design a diagnostic device capable to efficiently detect deviations from the normal operating conditions due to fault occurrences. To this purpose we will resort to a class of "Luenberger-like" residual generators having the following expression:

$$\begin{cases} \dot{\hat{x}}(t) = a(\hat{x}(t)) + b(\hat{x}(t))u(t) + L(\hat{x})\left(y(t) - h(\hat{x}(t))\right) \\ r(t) = h(x(t)) - h(\hat{x}(t)), \end{cases} \quad (2)$$

where $L(\hat{x}) \in \mathbb{R}^{n\times p}[\hat{x}]$ denotes the observer gain, which is supposed to be a matrix of multivariate polynomials in the state estimate $\hat{x}$. As standard in robust FD problems, the designed residual generator needs to meet suitable fault sensitivity requirements. This means that the filter must be capable of discriminating internal anomalous dynamic behavior due to faults from those pertaining to all other nuisances (disturbances and reference inputs).

Note that the objectives of robust residual generation are partially conflicting with each other, and there exists an unavoidable trade-off between the minimization of the disturbance effects and reference input on the residual and the maximization of the residual sensitivity to faults. Observe also that, for solvability reasons, the above sensitivity maximization makes it sense over a prescribed frequency range.
As will be clear in what follows, it is supposed that the exogenous/reference inputs and fault signals will not "overlap" in terms of frequency spectra. This means that, if the fault signal exhibits a relevant harmonic component at a given frequency, say $\bar{\omega}$, the disturbance spectrum, evaluated at the same frequency, will present instead a numerically negligible harmonic component. The converse (relevant harmonic components for the disturbance and negligible corresponding fault spectrum value at a given frequency) is also true. This requirement can then be carried out by shaping arbitrary exogenous/reference inputs and fault signals belonging to Ω_d, Ω_u, and Ω_f with *a priori* chosen linear filters whose finite-dimension realizations are characterized as follows;

$$\begin{cases} \dot{x}_d(t) = A_d x_d(t) + B_d d(t) \\ \hat{d}(t) = C_d x_d(t) + D_d d(t) \end{cases} \quad (3)$$

$$\begin{cases} \dot{x}_f(t) = A_f x_f(t) + B_f f(t) \\ \hat{f}(t) = C_f x_f(t) + D_f f(t). \end{cases} \quad (4)$$

Notice that the filter (3) will be also used to shape the reference input under the hypothesis that (see [13] for a detailed discussion