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Compensation of state-dependent state delay for nonlinear systems

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ABSTRACT

We extend the technique for compensating state-dependent delays from systems with delayed inputs to systems with delayed states. We focus on predictor-feedback design for nonlinear systems in the strict-feedback form, having a state-dependent state delay on the virtual input. The two key challenges are the definition of the predictor state and the fact that the predictor design does not follow immediately from the delay-free design. We resolve these challenges and we establish asymptotic stability of the resulting infinite-dimensional nonlinear system for general nonnegative-valued delay functions of the state. Due to an inherent limitation on the delay rate, and since the delay rate depends on the state, we obtain only regional stability results. However, for forward-complete systems, we provide an estimate of the region of attraction in the state space of the infinite-dimensional system. We finally provide two examples, including an example of stabilization of a cooling system.

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1. Introduction

State-dependent state delays appear in many engineering applications. Examples include milling processes [1], engine cooling systems [2], irrigation channels [3], network congestion control [4], population dynamics [5], supply networks [6,7] and automatic landing systems [8].

Predictor-based techniques are an indispensable part of the control design toolbox [9], for unstable linear plants with constant delays affecting the input [10–13] or simultaneously affecting inputs and states [14–17]. Various control schemes also exist for nonlinear systems with constant delays affecting the input [18–20] or state [21–23]. Yet, extensions of the predictor-feedback design to nonlinear systems with constant input delay had not been developed until recently [24,25]. Although in [26] (see also [27]), a predictor-based controller for unstable linear plants with time-varying input delay is developed, only recently a Lyapunov function was provided [28]. Finally, although nonlinear systems with simultaneous time-varying input and state delays are considered in [29], predictor-like designs for nonlinear systems with time-varying input delays [30] or simultaneous input and state delays [31] were developed recently. In [32], we introduced a technique for compensating state-dependent delays on the input of a nonlinear system. In this paper, we generalize this technique to systems that include state-dependent delays on the states of the system.

We consider forward-complete systems that are globally stabilizable in the absence of the delay. We then "backstep" one state-dependent integrator and design a predictor-based control law for the overall system, using prediction intervals that depend on the current value of the state (Section 2). Using an invertible infinite-dimensional backstepping transformation we derive explicit bounds for the norm of the closed-loop system. Due to the fundamental limitation of the allowable magnitude of the delay function's gradient (the control signal never reaches the plant if the delay rate is larger than one) we use these bounds to estimate the region of attraction of the proposed controller (Section 3). Two simulation examples illustrate the application of the control design (Sections 4 and 5).

Notation: we use the common definition of class $\mathcal{K}$, $\mathcal{K}_\infty$ and $\mathcal{K}\mathcal{L}$ functions from [33]. For an n-vector, the norm $|\cdot|$ denotes the usual Euclidean norm. We say that a function $\xi : \mathbb{R}_+ \times (0, 1) \mapsto \mathbb{R}_+$ belongs to class $\mathcal{K}\mathcal{C}$ if it is of class $\mathcal{K}$ with respect to its first argument for each value of its second argument and continuous with respect to its second argument. It belongs to class $\mathcal{K}\mathcal{C}_\infty$ if it is in $\mathcal{K}\mathcal{C}$ and also in $\mathcal{K}_\infty$ with respect to its first argument.

2. Problem formulation and controller design

We consider the following system

$$\dot{X}_1(t) = f_1\left(t, X_1(t), X_2\left(t - D\left(X_1(t)\right)\right)\right) \tag{1}$$

$$\dot{X}_2(t) = f_2\left(t, X_1(t), X_2(t)\right) + U(t), \tag{2}$$

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where $X_1 \in \mathbb{R}^n$, $X_2, U \in \mathbb{R}$ and $t \geq t_0 \geq 0$. We assume that $f_1 : [t_0, \infty) \times \mathbb{R}^{n+1} \mapsto \mathbb{R}^n$ is locally Lipschitz with $f_1(t, 0, 0) = 0$ for all $t \geq t_0$ and that there exists a class $\mathscr{K}_\infty$ function $\hat{\alpha}$ such that

$$|f_1(t, X_1, X_2)| \leq \hat{\alpha}(|X_1| + |X_2|), \quad \text{for all } t \geq t_0. \tag{3}$$

We further assume that $f_2 : [t_0, \infty) \times \mathbb{R}^{n+1} \mapsto \mathbb{R}$ is locally Lipschitz with respect to $(X_1, X_2) \in \mathbb{R}^{n+1}$ with $f_2(t, 0, 0) = 0$ for all $t \geq t_0$. The goal of the paper is to show that for (1), (2) there exist functions $P(\theta)$ and $\sigma(\theta)$, where $t - D(X_1(t)) \leq \theta \leq t$, such that the controller

$$U(t) = -f_2(t, X_1(t), X_2(t)) - c_2(X_2(t) - \kappa(\sigma(t), P_1(t))) + \frac{\frac{\partial \kappa(\sigma, P_1)}{\partial \sigma} + \frac{\partial \kappa(\sigma, P_1)}{\partial P_1} f_1(\sigma(t), P_1(t), X_2(t))}{1 - \nabla D(P_1(t)) f_1(\sigma(t), P_1(t), X_2(t))}, \tag{4}$$

where c_2 is an arbitrary positive constant and

$$P_1(\theta) = X_1(t) + \int_{t-D(X_1(t))}^{\theta} \frac{f_1(\sigma(s), P_1(s), X_2(s))\, ds}{1 - \nabla D(P_1(s)) f_1(\sigma(s), P_1(s), X_2(s))}, \quad t - D(X_1(t)) \leq \theta \leq t \tag{5}$$

$$\sigma(\theta) = \theta + D(P_1(\theta)), \quad t - D(X_1(t)) \leq \theta \leq t, \tag{6}$$

for $t \geq t_0$, compensates the state-dependent state delay and achieves asymptotic stability of the resulting closed-loop system. We refer to the quantity $P_1(\theta)$ given in (5) as "predictor" since $P_1(t)$ is the $D(P_1(t))$ time units ahead predictor of $X_1(t)$, i.e., $P_1(t) = X_1(t + D(P_1(t)))$. This fact can be seen as follows. Differentiating relation (5) with respect to θ and setting $\theta = t$ we get

$$\frac{dP_1(t)}{dt} = \frac{f_1(\sigma(t), P_1(t), X_2(t))}{1 - \nabla D(P_1(t)) f_1(\sigma(t), P_1(t), X_2(t))}. \tag{7}$$

Performing a change of variables $\tau = \sigma(t)$ in the ODE for $X_1(\tau)$ given by $\frac{dX_1(\tau)}{d\tau} = f_1(\tau, X_1(\tau), X_2(\tau - D(X_1(\tau))))$, we have that

$$\frac{dX_1(\sigma(t))}{dt} = \frac{d\sigma(t)}{dt} f_1(\sigma(t), X_1(\sigma(t)), X_2(t)). \tag{8}$$

From (8) one observes that $P_1(t)$ satisfies the same ODE in t as $X_1(\sigma(t))$ because from (6) to (8) it follows that

$$\frac{d\sigma(\theta)}{d\theta} = \frac{1}{1 - \nabla D(X_1(\sigma(\theta))) f_1(\sigma(\theta), X_1(\sigma(\theta)), X_2(\theta))}, \quad t - D(X_1(t)) \leq \theta \leq t, \tag{9}$$

provided that $P_1(t) = X_1(\sigma(t))$. Since from (5) for $t = t_0$ and $\theta = t_0 - D(X_1(t_0))$ it follows that $P_1(t_0 - D(X_1(t_0))) = X_1(t_0)$, by defining

$$\phi(t) = t - D(X_1(t)), \quad t \geq t_0, \tag{10}$$

$$\sigma(\theta) = \phi^{-1}(\theta), \quad t - D(X_1(t)) \leq \theta \leq t, \tag{11}$$

we have that $P_1(t_0) = X_1(\sigma(t_0))$. Noting from (10) and (11) that $D(X_1(\sigma(t))) = \sigma(t) - t$, differentiating this relation, we get (9). Comparing (7) with (8) we conclude with the help of (9) that indeed $P_1(t) = X_1(\sigma(t))$ for all $t \geq t_0$.

Motivated by the need to keep the denominator in (5) and (9) positive, throughout the paper we consider the condition on the solutions which is given by

$$\mathscr{G}_c : \nabla D(P_1(\theta)) f_1(\sigma(\theta), P_1(\theta), X_2(\theta)) < c, \quad \text{for all } \theta \geq t_0 - D(X_1(t_0)), \tag{12}$$

for $c \in (0, 1]$. We refer to $\mathscr{G}_1$ as the *feasibility condition* of the controller (4)–(5).

3. Stability analysis for forward-complete systems

Throughout the section, we make the following assumptions concerning the plant (1)–(2):

Assumption 1. $D \in C^1(\mathbb{R}^n; \mathbb{R}_+)$ and ∇D is locally Lipschitz.[1]

Assumption 2. There exist a smooth positive definite function R and class $\mathscr{K}_\infty$ functions α_1, α_2 and α_3 such that for the plant $\dot{X} = f_1(t, X, \omega)$, the following hold

$$\alpha_1(|X|) \leq R(t, X) \leq \alpha_2(|X|) \tag{13}$$

$$\frac{\partial R(t, X)}{\partial t} + \frac{\partial R(t, X)}{\partial X} f_1(t, X, \omega) \leq R(t, X) + \alpha_3(|\omega|), \tag{14}$$

for all $X, \omega \in \mathbb{R}^{n+1}$ and $t \geq t_0$.

Assumption 2 guarantees that the plant $\dot{X} = f_1(t, X, \omega)$ with ω as input is forward-complete.

Assumption 3. There exist functions $\kappa \in C^1([t_0, \infty) \times \mathbb{R}^n; \mathbb{R})$ and $\hat{\rho} \in \mathscr{K}_\infty$, such that the plant $\dot{X}(t) = f_1\big(t, X(t), \kappa\big(t, X(t)\big) + \omega(t)\big)$ is input-to-state stable with respect to ω and κ is uniformly bounded with respect to its first argument, that is,

$$|\kappa(t, X)| \leq \hat{\rho}(|X|), \quad \text{for all } t \geq t_0. \tag{15}$$

Theorem 1. *Consider the plant (1)–(2) together with the control law (4)–(6). Under Assumptions 1–3, there exist a class $\mathscr{K}$ function ξ_{RoA}, a class $\mathscr{KL}$ function β and a class $\mathscr{KC}_\infty$ function ξ_1 such that for all initial conditions for which X_2 is locally Lipschitz on the interval $[t_0 - D(X_1(t_0)), t_0]$ and which satisfy*

$$|X_1(t_0)| + \sup_{t_0 - D(X_1(t_0)) \leq \theta \leq t_0} |X_2(\theta)| < \xi_{\mathrm{RoA}}(c), \tag{16}$$

for some $0 < c < 1$, there exists a unique solution to the closed-loop system with $X_1 \in C^1[t_0, \infty)$, $X_2 \in C^1(t_0, \infty)$, and

$$|X_1(t)| + \sup_{t - D(X_1(t)) \leq \theta \leq t} |X_2(\theta)| \leq \beta\left(\xi_1\left(|X_1(t_0)| + \sup_{t_0 - D(X_1(t_0)) \leq \theta \leq t_0} |X_2(\theta)|, c\right), t - t_0\right), \tag{17}$$

for all $t \geq t_0$. Furthermore, there exists a class $\mathscr{K}$ function δ^*, such that for all $t \geq t_0$ the following holds

$$D(X_1(t)) \leq D(0) + \delta^*(c) \tag{18}$$

$$\left|\dot{D}(X_1(t))\right| \leq c. \tag{19}$$

The proof of Theorem 1 is based on Lemmas 1–8 which are presented next.

Lemma 1 (*Backstepping Transform of the Delayed State*)**.** *The infinite-dimensional backstepping transformation of the state X_2 defined by*

$$Z_2(\theta) = X_2(\theta) - \kappa(\sigma(\theta), P_1(\theta)), \quad t - D(X_1(t)) \leq \theta \leq t, \tag{20}$$

together with the predictor-based control law given in relations (4)–(5) transform system (1)–(2) to the "target system" given by

$$\dot{X}_1(t) = f_1(t, X_1(t), \kappa(t, X_1(t)) + Z_2(t - D(X_1(t)))) \tag{21}$$

$$\dot{Z}_2(t) = -c_2 Z_2(t). \tag{22}$$

[1] To ensure uniqueness of solutions.

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