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Wave spectral modeling of multidirectional random waves in a harbor through combination of boundary integral of Helmholtz equation with Chebyshev point discretization

Prashant Kumar [a], Huai Zhang [b,*], Kwang Ik Kim [a], Yaolin Shi [b], David A. Yuen [c]

[a] Department of Mathematics, Pohang University of Science and Technology, Pohang 790-784, South Korea
[b] Key Laboratory of Computational Geodynamics, University of Chinese Academy of Sciences, Beijing 100049, PR China
[c] Department of Geology and Geophysics, University of Minnesota, Minneapolis, MN 55455, USA

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ABSTRACT

A mathematical model is presented to predict the wave spectrum in a complex geometry harbor domain due to the diffraction of multidirectional random waves propagating through entrance of the harbor from the open sea. An accurate description of the harbor geometry, bathymetry and an incident wave spectrum are required. The solution of Helmholtz equation in the bounded and unbounded region is obtained by 2-D boundary integral method with the consideration of partial reflection and corner point approximation. Moreover, the Chebyshev point discretization is applied on boundary of the harbor to improve the accuracy of the numerical scheme. The principle of superposition is adopted to simulate the directional random waves based on the solution of monochromatic incident waves. The form of Mitsuyasu's frequency spectrum was utilized for the random wave simulation. The wave spectra for multidirectional random waves are analyzed at various recorder stations in the Pohang New Harbor (PNH). Comparison of simulation results with in situ measurement data recorded at various recorder observatories in the PNH demonstrates the feasibility and accuracy of our proposed method. After this study, we analyzed the wave height distribution in the whole PNH domain. Based on the simulation results, some tactic has been introduced to reduce the wave amplitude in the PNH. Our primary results confirm that present numerical model is accurate and flexible to implement on various real harbors to predict the multidirectional random wave diffraction.

1. Introduction

For many years, numerous wave spectrum models have been proposed in literature to calculate the wave field in a harbor, and they bring forward results to protect the coastal structures, such as harbor, bays, and breakwaters, under harsh seasonal weather conditions [1–3]. The study of harbor resonance is an equivocal area in the coastal research, and a large number of research papers, reports and thesis have been contributed to understand the various aspect of this problem (e.g. Caudal [4]; Gagnaire-Renou et al. [5] and Dongeren et al. [3]). Wave induced oscillation in an arbitrary shaped harbor based on Helmholtz equation is analyzed [6–8].

The wave spectrum model, either analytical or numerical, must be adequately chosen to draw the reliable blueprint of the coastal structure in construction. Essentially, the amplitude of the harbor oscillation could become large at resonance frequencies due to diffraction and partial reflection of directional random waves from the interior boundary of the harbor. These oscillations could create serious problem on the moored ship itself or excessive mooring force leading to break the mooring ropes. For instance, Pohang New Harbor (PNH), which is situated in the southeast part of South Korea, has experienced extreme wave hazards during the serious seasonal weather conditions.

Several researcher as Nagai [9], Goda et al. [10] and Gaillard [11,12] used linear superposition method to determine the random wave diffraction inside the harbor. Firstly, the partial refection on the harbor boundary has been introduced by Isaacson and Qu [13]. Further partially reflecting boundary was used by various authors such as Chou and Lin [14] and Chou and Han [15,16] to monochromatic response of linear waves. An experiment was conducted by Girolamo [17] to analyze the harbor resonance induced by regular and irregular incident waves. The combination of incident wave absorber boundary and partial reflection boundary

* Corresponding author.
E-mail addresses: iitd.prashant@gmail.com (P. Kumar), hzhang@ucas.ac.cn (H. Zhang), kimki@postech.ac.kr (K.I. Kim), shiyl@ucas.ac.cn (Y. Shi), daveyuen@gmail.com (D.A. Yuen).

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conditions are studied by Hamanaka [18] to analyze the harbor resonance. Bellotti [19] used the combination of Helmholtz and mild slope equation to analyze the transient response of long waves under the resonance frequencies in a harbor.

A mathematical model was developed by Berkhoff [20,21] to analyze phenomenon of combined diffraction and refraction of water waves. Several researchers Goda et al. [22], Mitsuyasu [23], Mitsuyasu et al. [24], Goda and Suzuki [25] have developed the mathematical models based on diffraction and refraction of random wave. The boundary element modeling based on diffraction of multidirectional random waves in an arbitrary shape harbor with partially reflecting boundaries have been presented by Lee and Williams [26], while Lee et al. [27,28] have used the same boundary element model for the rectangular submarine pits and the navigator pit through rectangular channel, respectively. The comparison studies of the several wave spectra for the random wave diffraction by semi-infinite breakwater is done by Lee and Kim [29]. Overall, the concept of random wave diffraction in coastal engineering is reviewed by Goda [30].

In this paper, we analyzed the wave spectrum of multidirectional random waves in a realistic harbor with partially reflecting boundary including the corner contribution by using the principle of super position and Mitsuyasu's spectrum. The responses of monochromatic incident waves are estimated with partial reflection on the harbor boundary through the combination of boundary integral solution of Helmholtz equation with Chebyshev point discretization [31]. Our study consists of two parts. Firstly, the linear response of incident wave spectrum in the harbor has been analyzed under the assumptions of irrotational, incompressible and inviscid fluid, partial reflection and wave absorbing boundary. Secondly, the wave spectra are numerically estimated under the resonance frequencies with the consideration of multidirectional random waves by using the linear superposition and Mitsuyasu's spectrum. Further, the validation of present numerical model is accomplished by comparing the simulated results with real time in situ measurement data recorded at various recorder stations inside the PNH. The diffraction coefficient is computed under the resonance condition for the different directional wave spectrum in the interior of PNH. Based on the simulation results, the tactic for the improvement such as plant the wave absorbers at the entrance and at the breakwaters in the PNH is proposed. In results, the wave amplitude inside the PNH has been significantly reduced after the tactic implant in the PNH. Hence the current numerical model can easily implemented on realistic harbors, and is very useful for the hazard prevention and mitigation, or to protect the coastal structures based on various aspects of harbor resonance.

2. The mathematical model

The domain of interest is divided into two regions, as shown in Fig. 1. The first one is the bounded region Ω_b (or harbor region), which is surrounded by the partially reflecting harbor wall S_W and by the pseudo boundary Γ_0. The bounded region Ω_b includes the harbor entrance E_1E_2 and pseudo boundary Γ_0. Another one is the unbounded region Ω_u, which is attached to the bounded region by the common pseudo boundary Γ_0 (see Fig. 1). The origin of the Cartesian coordinates is located arbitrarily on the entrance E_1E_2, x-axis is placed along the shoreline at the entrance, y-axis is directed towards the open sea and z-axis is directed vertically upwards from the sea surface. The incident waves diffract at the entrance corners, and partially reflect from the interior harbor boundary S_W with sharp corner of angle θ (see Fig. 1).

We assumed that the fluid's flow is irrotational, and the fluid is considered inviscid and incompressible. The velocity vector $\vec{u}$ is expressed as the gradient of the velocity potential Φ, i.e., $\vec{u} = \nabla\Phi$

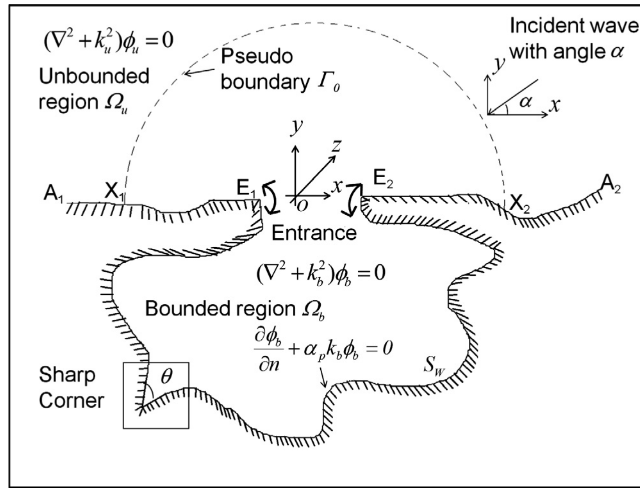

Fig. 1. Model based harbor domain with boundary condition due to the partial reflection on the harbor walls S_W.

where $\vec{u}$ has u, v and w components in the x, y and z direction, respectively. The velocity potential is determined by the following expression

$$\Phi(x,y,z,t) = -\frac{A_0 g}{i\omega}\frac{\cosh(k(z+h))}{\cosh kh}\phi(x,y)\exp(-i\omega t), \tag{2.1}$$

where A_0 is the amplitude of an incident wave, g is the gravitational acceleration, h is water depth and wave number k is described in terms of angular wave frequency ω from dispersion relation $\omega^2 = gk\tanh kh$. The wave function ϕ must satisfy the Helmholtz equation $(\nabla^2 + k^2)\phi = 0$, since $\nabla\cdot\vec{u}$ is zero for incompressible fluid.

2.1. Wave function in the bounded region

In order to analyze the velocity potential in the bounded region, the major concern is to determine the wave function $\phi_b(x,y)$ which satisfies the $(\nabla^2 + k_b^2)\phi_b = 0$. It is solved in the bounded region Ω_b by utilizing the Green's second identity. The zeroth order Hankel function [32] of the first kind $H_0^{(1)}(kr)$ is chosen to be the fundamental solution of the 2-D Helmholtz equation, which possesses the singularity at the origin as kr approaches to zero. The wave function $\phi_b(x,y)$ at any point (x,y) inside the harbor is expressed by the following integral form

$$\phi_b(x,y) = C\int_{\Gamma_0\cup S_W}\left\{\phi_b(x_0,y_0)\frac{\partial}{\partial n}H_0^{(1)}(k_b r) - H_0^{(1)}(k_b r)\frac{\partial}{\partial n}\phi_b(x_0,y_0)\right\}ds(x_0,y_0), \tag{2.2}$$

where (x,y) is the interior point of the harbor, (x_0,y_0) is the integration variable on the harbor boundary $\Gamma_0\cup S_W$ and the distance between the point of consideration (x,y) to boundary point (x_0,y_0) $r = \sqrt{(x-x_0)^2 + (y-y_0)^2}$. The complex number parameter C is given as follows:

$$C = \begin{cases} -i/4 & \text{if } (x,y)\in\Omega_b, \\ -i/2 & \text{if } (x,y)\in\Gamma_0\cup S_W, \\ -i\pi/2\theta & \text{if } (x,y) \text{ at the corner point,} \end{cases} \tag{2.3}$$

where θ is the corner angle. To determine the wave function ϕ_b specifically in the bounded region Ω_b, which is denoted by ϕ in Eq. (2.1), we have to consider the following three cases which have different kind of boundary conditions:

(a) Bottom boundary condition on the sea floor as fluid flow does not penetrate the sea bed is written as follows: